Calculating capital requirements for longevity risk in life insurance products. Using an internal model in line with Solvency II

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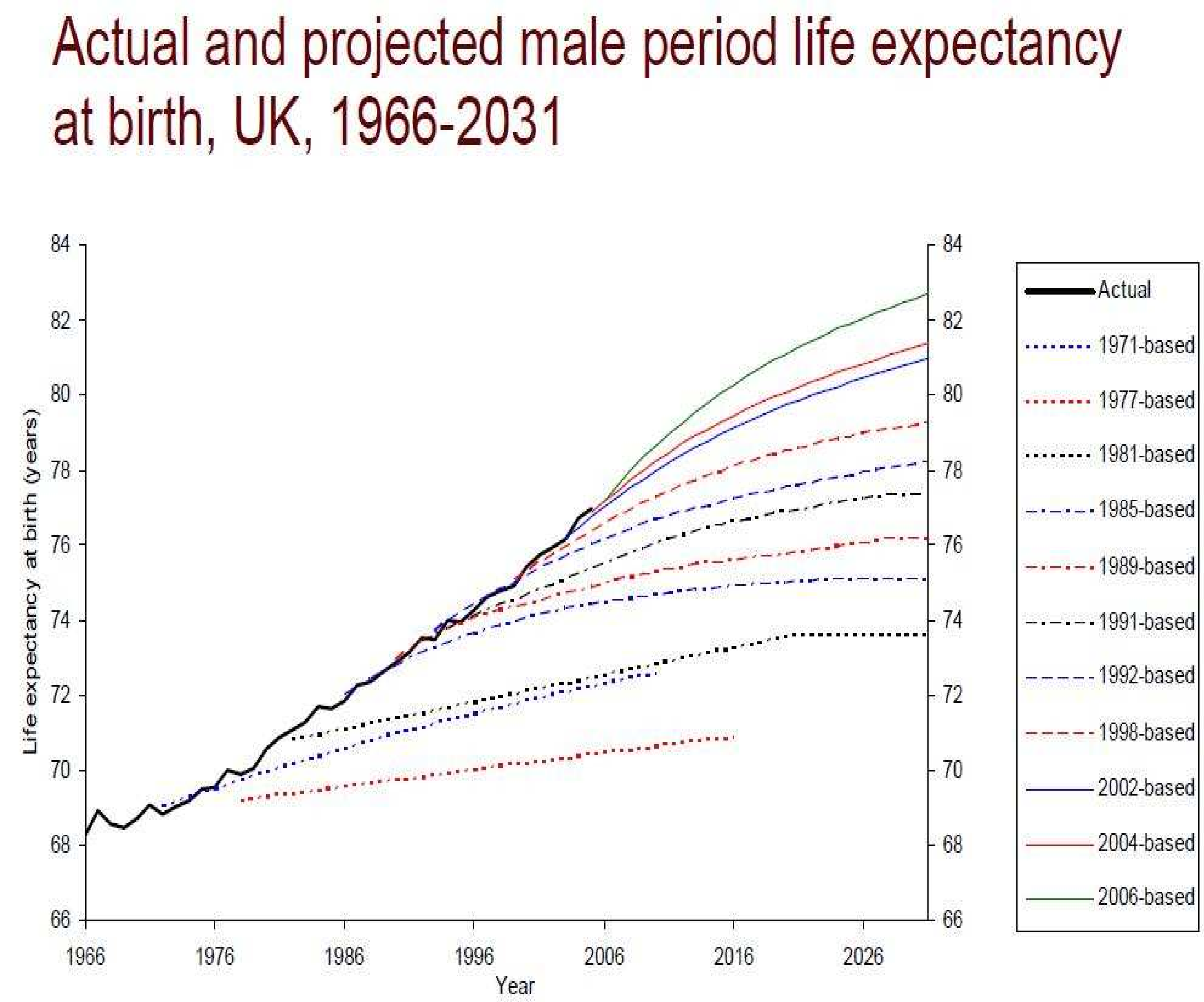
Introduction

Outline of presentation

- Introduction
- Solvency II regulations
- Approximation
- Results
- Conclusions


Actual and projected male period life expectancy at birth, UK, 1966-2031
Life expectancy at birth (years)
Year
1966 1976 1986 1996 2006 2016 2026
66 68 70 72 74 76 78 80 82 84
Actual
1971-based
1977-based
1981-based
1985-based
1989-based
1991-based
1992-based
1998-based
2002-based
2004-based
2006-based

Research question

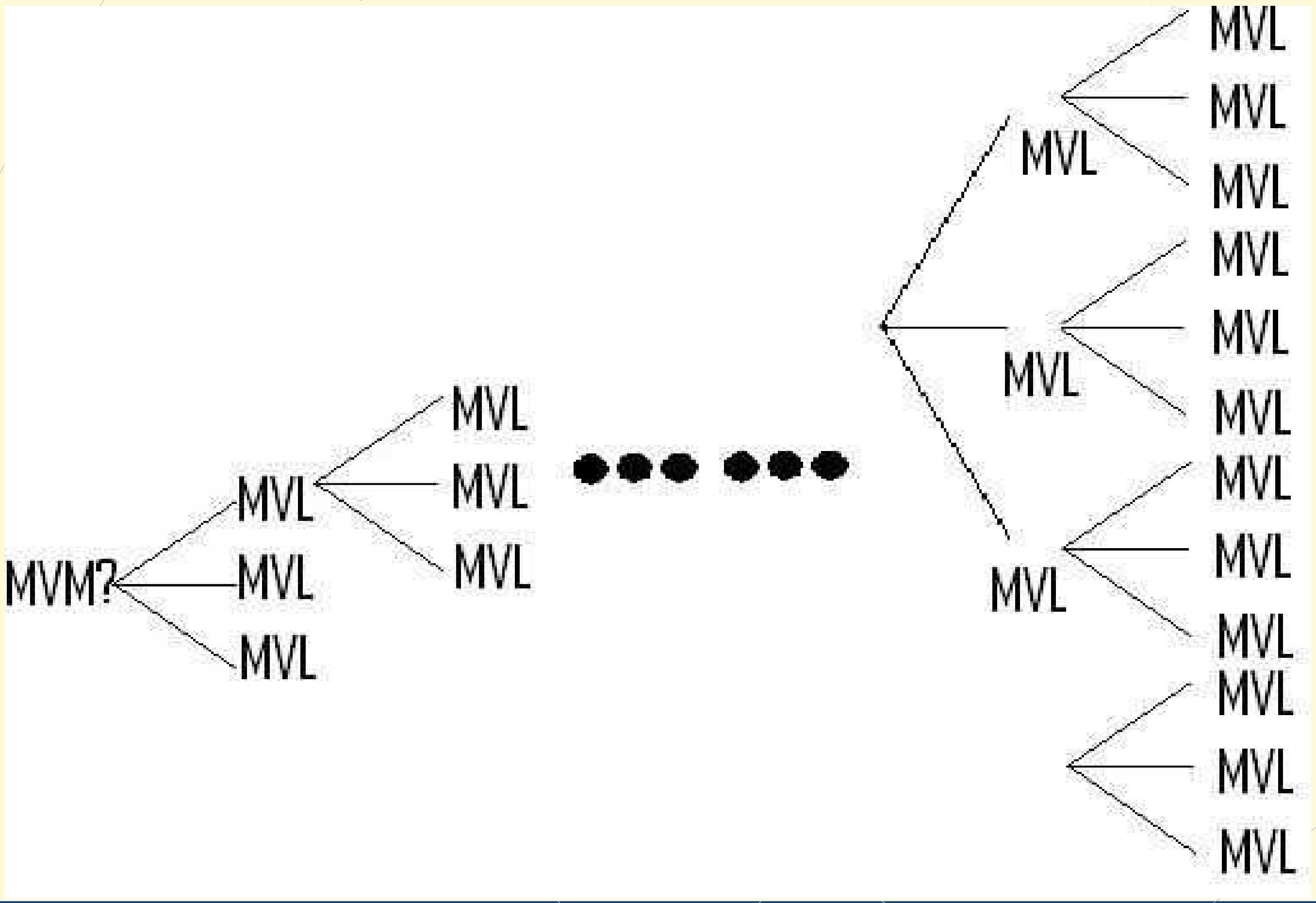
- What is the amount of capital an insurer should hold in order to set the probability of underfunding within a year to 0.5%?
- Requires joint distribution of:
  - The stochastic return on assets;
  - The stochastic payments made between now and next year;
  - The stochastic fair value of the liabilities now/next year, i.e., the fair value = BEL (Best Estimate of the Liabilities) + MVM (Market Value Margin).
- Hence, capital requirements today depend on fair value next year.
- Fair value next year (and payments) is stochastic, depending on the evolution of survivor probabilities!

Problem I

- Drawback of mortality forecast models:
  No closed form expression;
- We derive a closed form approximation of the distribution using Lee Carter-model;
  - Use the approximate distribution for solvency capital requirements;
  - Approximate distribution $\longrightarrow$ no nested simulations;
- Idea of CoC-approach (for a 1-year risk):
  - Insurer should hold a reserve (SCR);
  - Reserve $\longrightarrow$ lower return than free assets
    $\longrightarrow$ compensation: a Cost of Capital rate (CoC) multiplied by the SCR;
  - Market Value Margin: MVM = CoC% $\cdot$ SCR.

Problem II

- MVM now depends on distribution MVL next year.
- MVM in year before last payment: simulate distribution of MVL in last year for each scenario.



Solvency II regulations

- Solvency II:
  - Capital requirements based on funding ratio approach, with $\mathbb{P}$(underfunding)$< 0.5\%$;
  - Close to the Swiss Solvency Test (SST);
  - Longevity risk has a market price;
  - Road map: implementation in 2013.
- Insurer can use:
  - Simplified approach (Solvency II);
  - Internal model.

Simplified approach

- Difficult to determine future buffers $SCR_{t+\tau}$
- Simplified approach:
  - SCR: net change in asset minus liabilities under two scenarios, namely:
    - i) best estimate scenario;
    - ii) longevity shock scenario (mortality probabilities $\downarrow$ 25%);
  - Future SCR: Use scenarios conditional on current information.
- Market Value Margin: Cost of Capital rate $\cdot \sum$ discounted SCR's.

Funding ratio approach

- First consider an one-year risk.
- Funding ratio approach:

$$\text{Funding Ratio} = \frac{\text{Assets}}{\text{(Fair value of) liabilities}}$$

- Probability of underfunding:

$$\mathbb{P}_t \left(FR_{t+1} < 1 \right) = \mathbb{P}_t \left(A_{t+1} - L_{t+1} < 0 \right).$$

- Market value liabilities:

$$L := BEL + MVM.$$

- Total capital requirement:

$$A^\star := BEL + MVM + SCR.$$

- Market price of risk (r: risk-free return):

$$MVM_t = \sum_{\tau \geqslant 0} \frac{(r + CoC) \cdot SCR_{t+\tau}}{(1+r)^\tau}.$$

- Life insurance products typically have long run-off.
- One period model, sell liabilities after one year:

$$A_t^\star = L_t + SCR_t := Q_{t,0.995}\left[\frac{\widetilde{L}_t + L_{t+1}}{1+r}\right]$$

- Then the market value of the liabilities is:

$$L_t = BEL_t^{1period} + (CoC + r)\cdot SCR_t$$

$$BEL_t^{1period} = \mathbb{E}_t\left[\frac{\widetilde{L}_t + L_{t+1}}{1+r}\right]$$

- Equivalent MVM_t;

$$\mathbb{E}_t\left[\sum_{\tau\geqslant 0}\frac{CoC+r}{1+CoC+r}\cdot\frac{\left(\mathbb{Q}_{0.995,t+\tau}\left[\frac{\widetilde{L}_{t+\tau}+L_{t+\tau+1}}{1+r}\right] - L_{t+\tau}\right)}{(1+r)^\tau}\right].$$

Approximation

- Let $\mu^g_{x,\tau}$ be the force of mortality;
  We have (under appropriate assumptions):
$$\widehat{p}^g_{x,t+T} = \exp(-\widehat{\mu}^g_{x,t+T}).$$
- The Lee-Carter model:
$$\log(\mu^g_{x,\tau}) = a^g_x + b^g_x k^g_\tau + \epsilon^g_{x,\tau}.$$
- Extrapolate k^g_{t+T} using ARIMA(0,1,0):
$$k^g_{t+T} = k^g_{t+T-1} + c^g + e^g_{t+T}.$$
- To prevent jump off bias $\rightarrow k^g_t = 0$.

Forecast distribution

- We forecast the distribution of future forces of mortality:

$$\mu^g_{x,t+T} = \mu^g_{x,t} \cdot \exp\left(b^g_x T c^g + b^g_x \sum_{s=1}^{T} e^g_{t+s} + \epsilon^g_{x,t+T} \right).$$

- Three sources of risk
  - ϵ^g_{t+T} normally distributed;
  - e^g_{t+T} normally distributed;
  - c^g normally distributed;

- $\mu^g_{x,t+T}$ has a lognormal distribution;
- Variables of interest are nonlinear functions of the forces of mortality.

Approximations

- Let X be a $\log N$ variable with not too large uncertainty.

- To derive a closed form expression we make three approximations:
  - $X \sim \log N$ distributed variable is approximately $X \sim N$ distributed;
  - If $X \sim \log N$ distributed variable, then $1 - X \sim \log N$ distributed;
  - The sum of $\log N$ distributed variable is approximately $\log N$ distributed;

- Using Fenton & Wilkonson rule: the sum of lognormals is approximately lognormal;
  Dufresne (2004) shows that asymptotically for $\Sigma \to 0$ this is the case.

Market value liabilities

- At time $t + \tau - 1$: payments at time $t + \tau$ are lognormally distributed;
- At time $t + \tau - 2$: Market value (MVL) at time $t + \tau - 1$ is lognormally distributed;
- At time $t + \tau - 2$: Use Fenton & Wilkonson rule to obtain the distribution of the payments plus MVL.
  - at time s: MVL at time $s + 1$ depends on survival probabilities at time s;
  - Hence, MVL depends on the “updated” parameters of the survival probabilities;
  - At time s the “updated” value of the parameters at time $s + 1$ is normally distributed.
- Indication error: difference in MVM in best estimate scenarios between simulations and approximation: 1.25% NL males, 0.51% NL females.

Results

- Recall: idea of CoC-approach:
  - Holder of risk should hold a reserve;
  - Reserve leads to lower return than free assets $\rightarrow$ compensation for the reserve requirement;
  - MVM = CoC% $\cdot$ SCR (for one-year risk);
- Interest rate fixed at 4%;
- We consider four different portfolios;

M65: Male old-age pension aged 65;
F65: Female old-age pension aged 65;
H65: 0.5 male, 0.5 female old-age pension aged 65;
OA+PP: 0.5 male, 0.5 female old-age pension aged 65
0.35 male, 0.35 female partner pension.

- Using Cost of Capital rates of 4% and 6% in excess of the riskfree rate.

Drawback Solvency II simplifications

- Recall: Simplifications set capital requirements set using two scenarios;
- Drawbacks:
  - Unsuitable for capturing risks which slowly evolve over long period of time;
  - Shock does not depend on policy duration and age;
  - Longevity risk is assumed to be independent of gender;
  - The percentage reduction is very large.
- Capital requirements for CoC=6%, US:

A_1	$\frac{A^\star - BEL}{BEL}$	$\frac{MVM}{BEL}$	$\frac{SCR}{BEL}$
M65	20.59%	10.66%	9.93%
F65	17.12%	9.28%	7.84%
H65	18.75%	9.93%	8.83%
OA+PP	16.47%	9.33%	7.13%

- Other simplifications for calculating the MVM:

  A_2: Using the best estimate scenario instead of mean:

$$\widetilde{SCR}_{t+\tau} = \mathbb{Q}^{BE(t)}_{1-\alpha,t+\tau}\left[\frac{\widetilde{L}_{t+\tau} + BEL^{BE(t+\tau+1)}_{t+\tau+1}}{1+r^{rf}}\right] - BEL^{BE(t)}_{t+\tau}$$

  A_3: Assuming constant SCR relative to BEL:

$$\frac{\widetilde{SCR}_{t+\tau}}{SCR_t} = \frac{\mathbb{E}_t\left[BEL_{t+\tau}\right]}{BEL_t}$$

- Advantage: can reduce significantly computation time.

$$MVM_t = \sum_{\tau \geqslant 0} \frac{(r + CoC) \cdot \widetilde{SCR}_{t+\tau}}{(1+r)^{\tau}}.$$

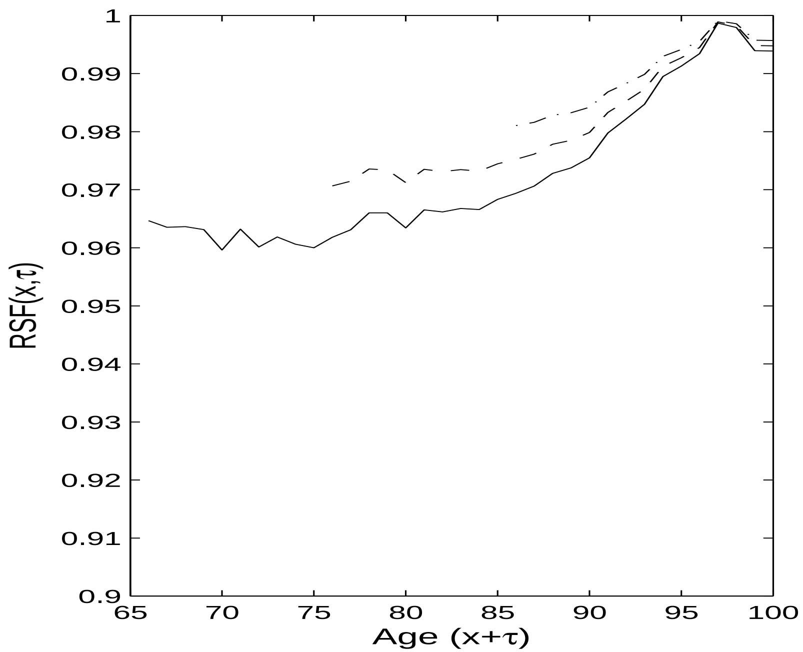
Internal model

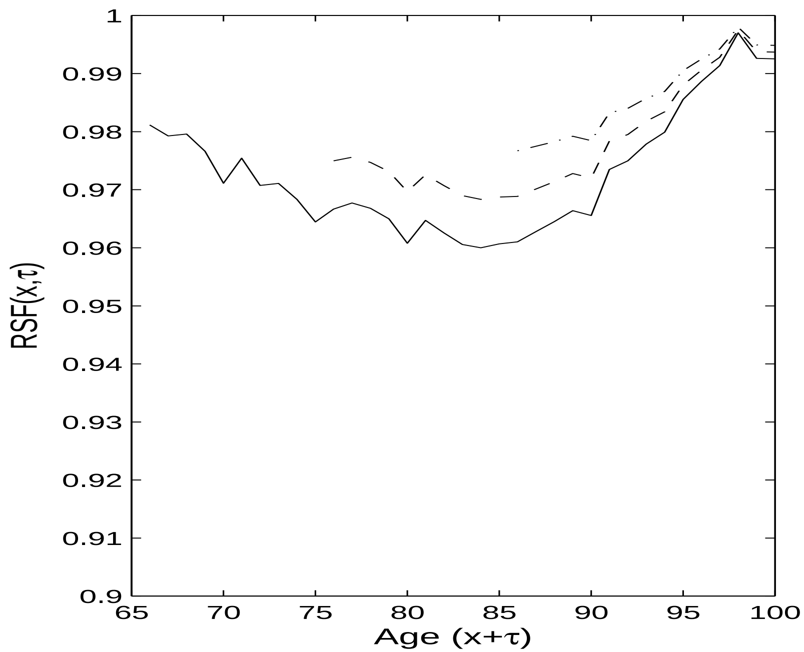
	Internal model			A_2	A_3
	$\frac{A^\star}{BEL}$	$\frac{MVM}{BEL}$	$\frac{SCR}{BEL}$	$\frac{MVM}{BEL}$	$\frac{MVM}{BEL}$
	US				
M65	2.08%	1.69%	0.43%	1.42%	1.14%
F65	1.63%	1.25%	0.39%	1.33%	1.06%
H65	1.64%	1.30%	0.36%	1.19%	0.95%
OA+PP	1.42%	1.21%	0.23%	1.12%	0.90%
	UK				
M65	3.53%	2.74%	0.86%	1.73%	1.38%
F65	3.70%	3.35%	0.45%	1.92%	1.54%
H65	3.19%	2.73%	0.53%	1.60%	1.28%
OA+PP	2.95%	2.64%	0.38%	1.52%	1.22%
	NL				
M65	2.28%	1.55%	0.75%	1.57%	1.26%
F65	4.32%	3.89%	0.58%	2.60%	2.08%
H65	2.93%	2.51%	0.48%	1.67%	1.34%
OA+PP	2.73%	2.45%	0.34%	1.69%	1.35%

Mortality uncertainty

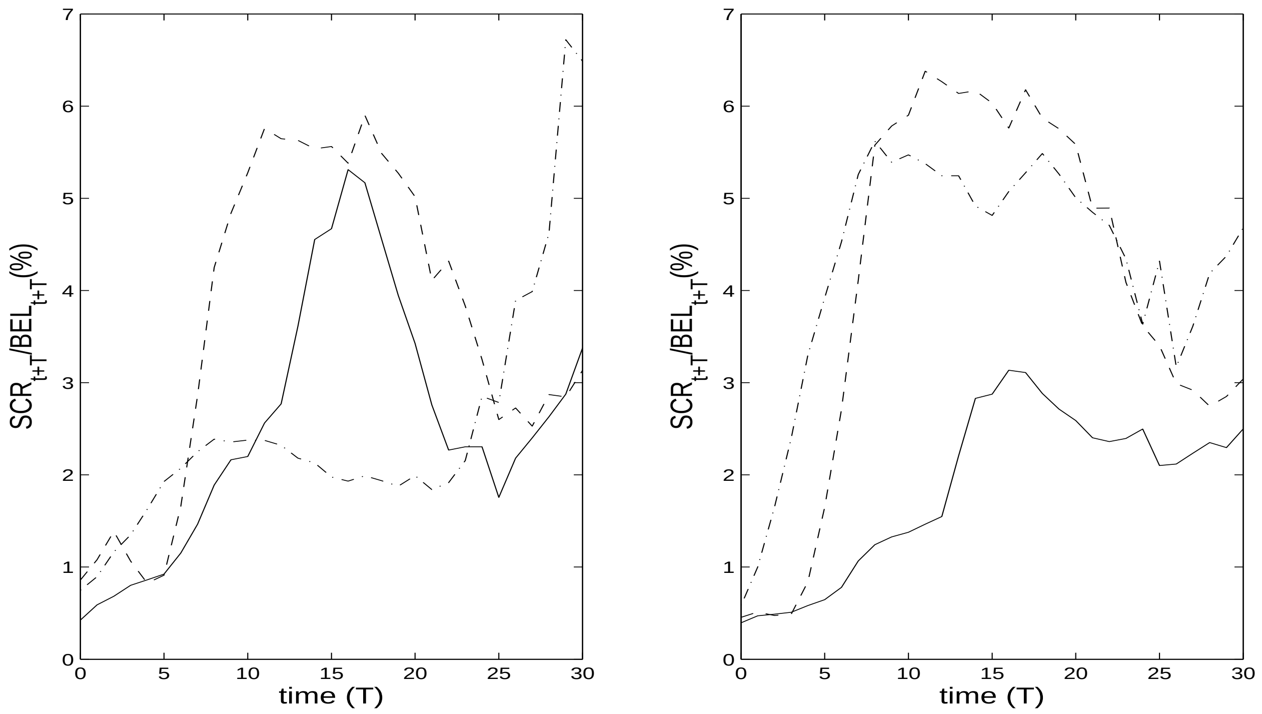
- Uncertainty in past mortality data indicate that jump of 25% in mortality probabilities is large;
- RSF: the relative shock factor in mortality probabilities in the 99.5% scenario relative to the best estimate scenario:

$$RSF(x,\tau) = \frac{\mathbb{Q}_{t,0.005}\left[q_{x+\tau,t+\tau}^{BE(t+1)}\right]}{q_{x+\tau,t+\tau}^{BE(t)}}$$





SCR over time



- Buffer (SCR) is non-monotonic function of time;
- Not a good approximation to calculate MVM using a constant SCR/BEL over time.

Conclusion

Conclusions

- We derived a closed form approximation for the distribution of longevity risk of variables of interest for life insurance liabilities;
- Solvency II simplification seems to be conservative;
- Calculate capital requirements using CoC-approach:
  - Makes it computational tractable;
  - Assumption of a constant SCR/BEL over time does not hold;
  - Best estimate scenario underestimates MVM.