Improving Longevity and Mortality Risk Models using Common Stochastic Long-Run Trends

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Introduction

- Last century:
  - Changes in causes of death have been common in the developing economies

    $\rightarrow$ common stochastic trends across countries

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    → common stochastic trends within a country across ages
- Causes of death = competing risks

→ How should we include the dependance in the modeling process?

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Data

Countries:

- Australia (1950 - 2003)
- Singapore (1963 - 2006)
- Japan (1950 - 2006)
- USA (1950 - 2005)
- Switzerland (1951 - 2005)
- Italy (1951 - 2002)
- Norway (1951 - 2005)
- Sweden (1951 - 2005)
- United Kingdom (1950 - 2006)

Data

Countries:

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Diseases:

- Diseases of the circulatory system
- Cancer
- Diseases of the respiratory system
- External causes (mainly: accidents)
- Infectious & parasitic diseases

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The model

VAR(p)

$$\mathbf{y}_t = \mathbf{c} + \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \mathbf{\Phi}_2 \mathbf{y}_{t-2} + \cdots + \mathbf{\Phi}_p \mathbf{y}_{t-p} + \epsilon_t$$

with

$$\begin{aligned} E(\epsilon_t) &= \mathbf{0} \\ E(\epsilon_t \epsilon_l) &= \begin{cases} \mathbf{\Omega} & \text{for } t = l \\ \mathbf{0} & \text{for } t \neq l \end{cases} \end{aligned}$$

$\rightarrow$ Need a stationary process

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$\rightarrow$ Need a stationary process

$\rightarrow$ What should we do with non-stationary variables?

Non-stationary variables

Possible answers

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- Work on the first difference

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- Use cointegrated relations

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  → Variables with common trends

Non-stationary variables

Possible answers

- Work on the first difference
- Use cointegrated relations
  - → Variables with common trends
  - → Long-run equilibrium relationships

Model selection

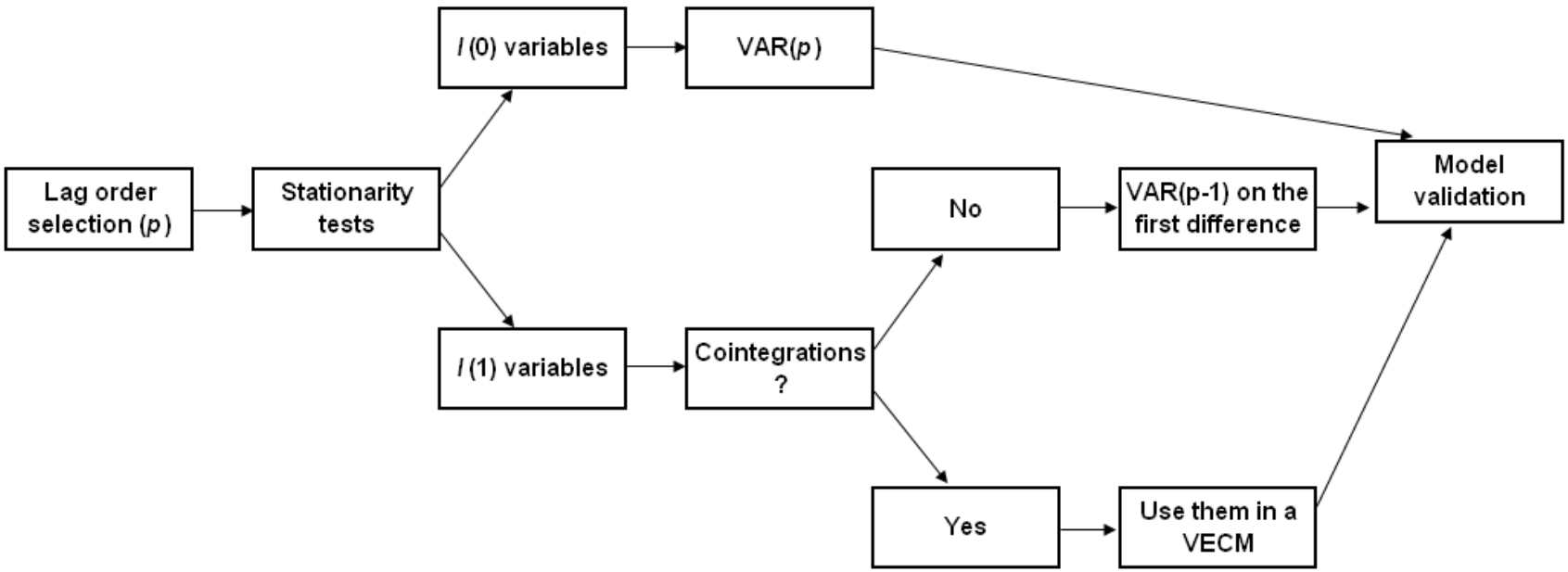

Figure: Steps to follow in a VECM analysis

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Lag order: 1

$\rightarrow$ VAR(1)

Stationarity test:

Lag order: 1

$\rightarrow$ VAR(1)

	Males	Females
USA	All causes: UR	All causes: UR
Australia	All causes: UR	All causes: UR
Switzerland	Respiratory: S Other causes: UR	Cancer, Respiratory: S Other causes: UR
Japan	Respiratory: S Other causes: UR	Respiratory: S Other causes: UR
Singapore	I&P: S Other causes: UR	I&P: S Other causes: UR
Italy	Respiratory: S Other causes: UR	All causes: UR
Norway	Respiratory: S Other causes: UR	Respiratory: S Other causes: UR
Sweden	Respiratory: S Other causes: UR	All causes: UR
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Figure: Stationarity of the five main causes of death in nine countries

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Figure: Stationarity of the five main causes of death in nine countries

Cointegrated relations:

	Males	Females
USA	2	1
Australia	3	1
Switzerland	1	0
Japan	1	3
Singapore	0	1
Italy	1	1
Norway	1	1
Sweden	1	1
United Kingdom	2	1

Figure: Number of cointegrated relations among the five main causes of death in nine countries

Cointegrated relations:

		Circulatory system	Cancer	External causes	Infectious and parasitic diseases
Switzerland - Males		-14.90	17.52	17.78	-0.64
Japan	Males	0.99	-4.61	-3.33	1.20
	Females	-2.42	-27.85	21.31	-2.98
Italy - Males		8.42	15.32	-18.75	2.40
Norway - Males		-6.80	-8.81	4.31	-2.52
Sweden - Males		12.08	16.98	-13.21	3.88
UK - Females		-2.71	2.08	1.32	-1.53

Figure: Long-run equilibrium relationships, diseases of the respiratory system stationary

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Heligman-Pollard function

$$q_{x,t} = \underbrace{A_t^{(x+B_t)^{C_t}}}_{} + \underbrace{D_t e^{-E_t(\ln(x)-\ln(F_t))^2}}_{} + \underbrace{\frac{G_t H_t^x}{1 + K_t G_t H_t^x}}_{}$$

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$\rightarrow$ Mortality rates due to the diseases of the circulatory system for females in the USA

Procedure

1. Fit the Heligman-Pollard function over the period 1950-2000

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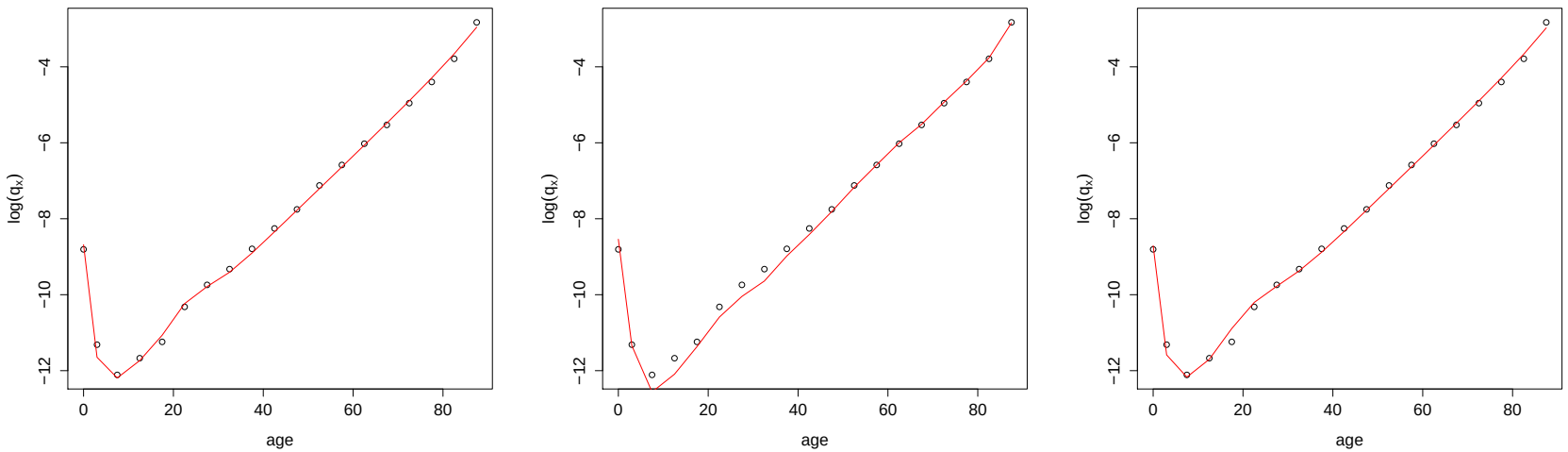
5. Compare the results with the actual data over 2001-2005

Results



(a) HP + VECM (b) Lee-Carter (c) HP + ARIMA

Figure: Log-mortality rates and the forecasted values, Circulatory system, USA, females, 2001

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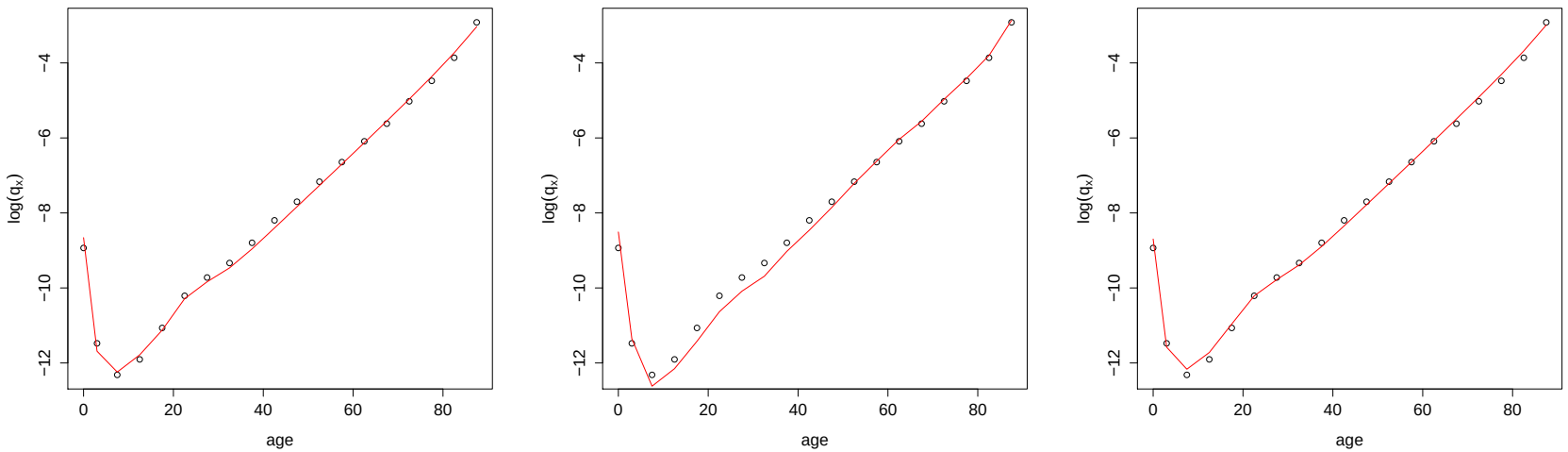
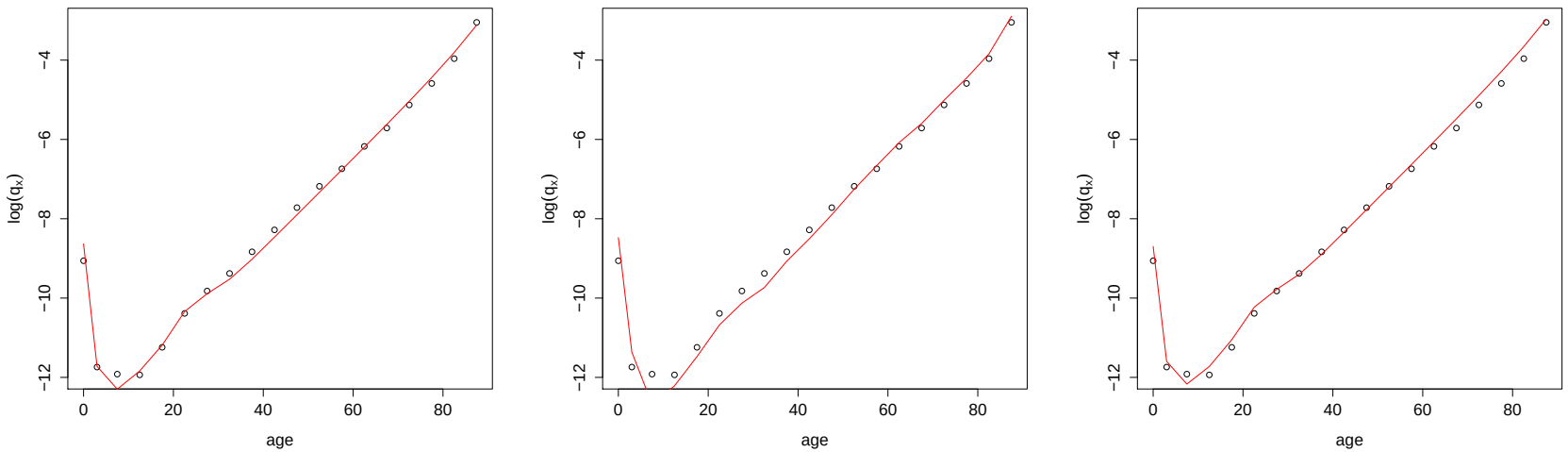


(a) HP + VECM (b) Lee-Carter (c) HP + ARIMA

Figure: Log-mortality rates and the forecasted values, Circulatory system, USA, females, 2003

Results



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Concluding remarks

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  - → Important considerations for a risk diversification approach

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→ New models incorporating this information need still to be developed

Thank you for your attention!