# Grandma and Grandpa take risks.

A study of the investment risks and longevity risks taken by Account Based pensioners in Australia.

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# Portfolio Choice, Longevity and Investment Returns

## Synopsis

In common with many countries experiencing an aging population and pronounced demographic shifts in population, Australia has been looking at various public policy responses to funding retirement. Australia, somewhat like the US (with the 401k plans) has adopted a largely Defined Contribution  system called 'superannuation' to provide funding for retirement, supplementing an existing means-tested 'age-pension' and individual savings. The superannuation available at retirement age is largely commuted to an Account Based pension – essentially a reverse defined contribution arrangement incurring investment risk.

This paper will examine the implications of longevity risk and of the investment choice in superannuation savings by individuals. This paper will not assess the interaction between superannuation and other forms of retirement funding.

**Are there benefits to a Defined Contribution System?** The imputed benefits of a Defined Contribution system include portability of savings, independence from employer for an increasingly mobile and flexible workforce, and of course ownership. The last aspect is becoming more evident with research by ASFA[1] showing that the majority of people are aware of superannuation and that once this becomes the largest asset outside the family home, investors

[1] Association of Superannuation Funds of Australia (ASFA) is one of a number of large and very influential industry bodies with highly respected research capabilities.

are more likely to take an interest. In aggregate, this Defined Contribution system has resulted in Australia having the 4th largest pool of investible assets in the world[2].

In contrast to Australia, other countries such as Norway, the Netherlands and Sweden have developed different systemic responses with significant Defined Benefit components run by the government. This is a common European response, where employers or individuals contribute to some national scheme as part of their employment conditions, and in turn receive some retirement benefits over and above those generally available via a basic age pension. Such systems are generally independent of the employer, and often offer additional capacity for individuals to save within the system through additional contributions. Seemingly, they provide much of the benefits of portability and employer independence with some community sharing of investment and longevity risk.

So if the key characteristic difference of the Defined Contribution system in Australia is that the individual bears the entire investment risk of their superannuation savings, what is the offsetting benefit? A key benefit often enunciated is that where individuals have a direct relationship with their retirement savings, as they do in a Defined Contribution system, they are more likely to be engaged in managing their investments. Mann (2009) argued that this shift of risk is about shifting the balance of state responsibility and individual rights and risks.

Lien (2007) also argues that in Norway, the societal association with the Norwegian Pension Fund is sufficient to enable Norwegians to have a relationship with their retirement funding, without the need for individual accounts.

[2] The Australian Government Publication : AXISS 2006

On a macroeconomic level, Nishiyama (2005) argued that the effects of privatising a portion of the pension system in small open economy (a description that probably fits Australia) led to a significant increase in overall wealth. Nishiyama includes the effects of a Social Security system operating to partially offset wages shock and longevity risk, and argues that "households re-optimize their lifecycle choices after the policy change" to produce some of the efficiency gains. So what are the risks faced by individuals in a Defined Contribution system?

**Post retirement.** Up until retirement, the argument that investors can readily incur the risk of choices between investment options because they are employed, and therefore are able to supplement or reduce their contributions to their retirement funds (within constraints). Furthermore, the capacity to access low cost life insurance (with legislated minimum levels) means that the individual has the capacity to insure against early death.

Post retirement however, the circumstances change significantly. Suddenly there is less or no capacity to make additional contributions to their retirement funds, implying that the retiree undertakes a more concentrated risk of their investment. Then over and above this, is the risk of longevity – in effect the opposite of the risk insured away pre-retirement. In this case, however there is no capability to insure away that risk.

**The Central Limit Theorem and dispersion of returns.** A common error made by media and analysts, is that risk reduces over a long period of time, and somehow there is more certainty in the terminal value of the investment. *"(P)ortfolio strategies that are uniformly the same in every period give rise to a cumulative sum of logarithms of returns that do approach, when normalized under specified conditions .. a Gaussian distribution" (Merton (1973)).* Of course Merton and Samuelson paper goes to on highlight the inconsistencies of the assumptions required to sustain this argument. Furthermore, Sharpe (1964) and Kritzman (2003) both highlight the effect of a distribution over time on expected terminal values. It seems then that a better understanding of the actual dispersion of returns over time will yield a more accurate reflection of risk take, and put pay to the presumption that holding a portfolio long enough means that risk reduce.

Utilising data from SuperRatings for superannuation returns over the last 5 years, the paper reviews the dispersion of returns from superannuation funds and specifically models the risk taken by each individual, who does not have access to portfolio insurance or longevity risk insurance. Such insurance is not readily or efficiently available in Australia, and Mitchell (2006) specifically reviewed the failure of markets to provide some sort of longevity risk insurance.

The paper then develops a model for an investor who survives to age 65 and wishes to fund retirement until their expected age at death of 83 (the ‘target date’) – being the mean age at death of their cohort. The model use Monte Carlo simulation for investment returns, to develop a likelihood that the investor either will run out of money before their target date for their portfolio, or indeed through natural variation of returns, will actually have sufficient money even though they have exceeded their anticipated lifespan.

The paper draws on this preliminary analysis to assess the experience of the individual investor, and assess the industry response.

Finally, the paper concludes that although pooled risk annuities should provide an alternative, the fact that it does not implies that the risks that individuals incur are more than the market is willing to bear, and perhaps government pooling is required.

# Method

## The Model Setup

**Portfolio Choice and the dispersion of returns.** Superannuation members have an increasing variety of options available to them in selecting the portfolios through which to invest their superannuation. Around half the employees allow their superannuation savings to be invested in the fund selected by their employer as a default fund.3 Furthermore, the employees invest in whatever investment option is the default option for that fund. In some cases, the portfolio may vary in accordance with the age of the employee (a process known generally as lifecycle investing or target date investing). Employees, who do not follow employer defaults, will often select funds and investment options, with information provided which details specified risk-return characteristics.

Over the last decade, there has been the parallel emergence of a number of organisations that operate to amalgamate and publish the performance of superannuation funds and their investment options. These organisations include ‘SuperRatings’, ‘Rainmaker’, ‘Dexxr’, and ‘RiceWarner’.

This section utilises figures published by ‘SuperRatings’ to show the dispersion of annualised 5 year returns from a number of large superannuation funds and then uses the returns of the ‘default’ investment option of the funds responding to the SuperRatings survey, to develop

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[3] The superannuation system in Australia defines superannuation contributions as contributions made by employers on behalf of employees and where there is no explicit choice made by the employee, the employer is obligated to remit their employee superannuation contributions to a default fund that is selected formally by an employer. The default fund is generally a ‘corporate fund’, an ‘industry fund’ a ‘master trust’ or a ‘retail fund’, each of which are structures the governance of what are in essence mutual funds.

a model of investor experiences at an individual level. In both cases, 5 year annualised return and annualised standard deviation numbers have been used, as available at 30 April 2007.

**The dispersion of portfolio returns.** Figure 1 below shows that the dispersion of annualised returns for all superannuation funds over $200m relative to their reported allocation to growth assets.  As expected, the returns increase as the portfolio shows a greater allocation to growth assets. However, as anticipated, the figure also show a marked increase in dispersion of returns as the proportion of assets allocated to growth assets approaches 100%. (It is worth noting that only 1.8% of assets by value are described as having a 100% allocation to 'Growth Assets').

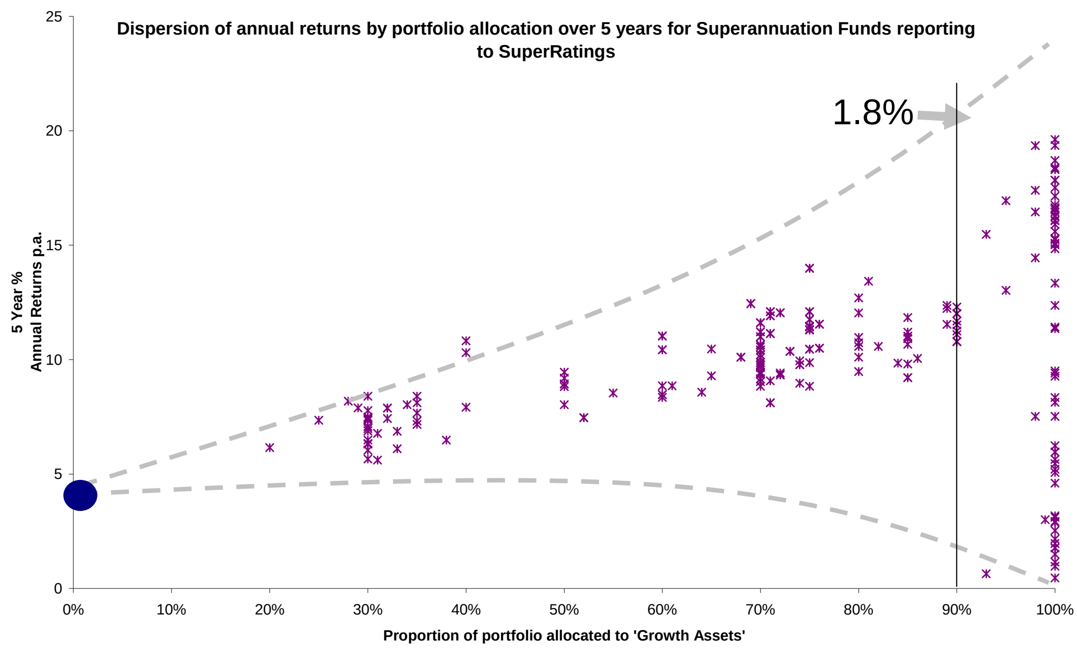


Figure 1 The dispersion of annualised returns versus the allocation to ‘Growth Assets’ over 5 years to April 2007 reporting to SuperRatings

Again, this is the expected pattern of dispersion. The large “Blue” marker at 0% allocation to ‘Growth Assets’ is the notional ‘risk-free’ using the returns on the ‘cash’ portfolio options of the funds.

An analysis of the annualised 5 year standard deviation of the portfolios compared to the returns is as shown in Figure 2 below:

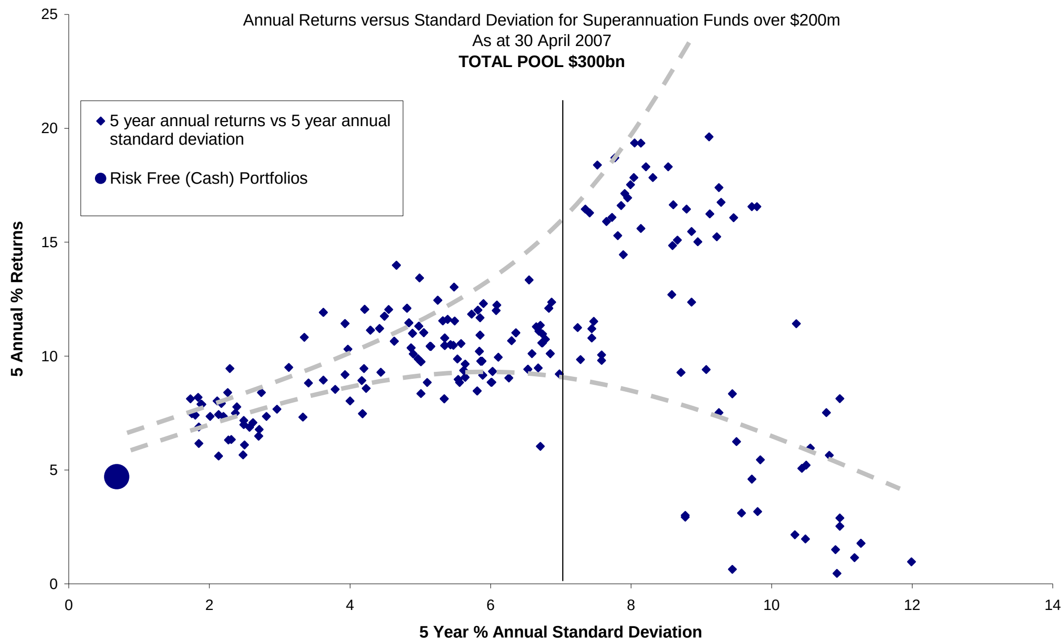


Figure 2 Annualised returns versus annualised standard deviation over 5 years to April 2007 for portfolios in superannuation funds reporting to SuperRatings

The dispersion is as expected, with returns broadly correlated to standard deviation of returns. It should be noted that just 2.8% of investments by asset value account for markers to the right of 7% annual standard deviation.

On an aggregate basis, therefore it can be argued that the pattern represented in figures 1 and 2 above, are broadly as predicted in theoretical models.

According to APRA, the majority of superannuation investors are passive in their initial choice with 17% of funds (by asset value) not offering investment choice, and with around 50% of members are in the default strategy[4].

Consequently, the returns in the 'default' portfolios are of great interest. The variation of returns for the chosen 'default' portfolio in the 'default' fund is shown below:

[4] APRA Annual Superannuation Bulletin June 2006 (Issued 22 March 2007) available from www.apra.gov.au

Table 1 Annualised Investment Returns for the nominated ‘Default’ Portfolio

| “SuperRatings” Fund Crediting Rate Survey - Default Options | | | | |
|---|---|---|---|---|
| Fund Investment Option | 5 Year Return | 10 Year Return | 5 Year Std Deviation | 5 Year Sharpe Ratio |
| Top Quartile | 10.3 | 10.1 | 4.7 | 1.1 |
| Bottom Quartile | 8.5 | 8.9 | 5.6 | 0.6 |
| All Fund Median | 9.1 | 9.3 | 5.2 | 0.9 |
| Maximum Value | 13.7 | 11.1 | 7.1 | 1.7 |
| Minimum Value | 6.4 | 7.3 | 3.0 | -1.1 |

The returns shown are consistent with a t-statistic that returns are not significantly different from those that can be explained by a normal distribution.

## The effect on individuals of the dispersion of portfolio returns

While the dispersion of aggregate returns is generally representative of those expected according to risk and return, the returns experienced by the individual correspond to just one return *matter*.

Assuming for the moment that the top and bottom quartile returns as shown in Table 1 above, are experienced by two superannuation investors, and assume that the annualised returns are distributed evenly and that the superannuation investors contribute $10,000 p.a. at the start of each year and have an opening balance of $30,000, the position at the end of April 2007 for these two investors are as shown in Table 2 below:

The table shows that the variation in returns at an individual level can significant, and that this may lead to behavioural responses that are different between investors that are invested in portfolios with the same aggregate risk-return characteristics.

Table 2 Comparison of Returns experienced by two investors, one of whom receives the top quartile of annual returns, the other who receives the bottom quartile of returns

| Top Quartile | Apr-03 | Apr-04 | Apr-05 | Apr-06 | Apr-07 |
|---|---|---|---|---|---|
| Opening Balance | 30,000 | 44,104 | 59,655 | 76,802 | 95,708 |
| Contributions | 10,000 | 10,000 | 10,000 | 10,000 | 10,000 |
| Returns | 4,104 | 5,551 | 7,147 | 8,906 | 10,846 |
| Closing Balance | 44,104 | 59,655 | 76,802 | 95,708 | 116,553 |

| Bottom Quartile | Apr-03 | Apr-04 | Apr-05 | Apr-06 | Apr-07 |
|---|---|---|---|---|---|
| Opening Balance | 30,000 | 43,392 | 57,920 | 73,679 | 90,775 |
| Contributions | 10,000 | 10,000 | 10,000 | 10,000 | 10,000 |
| Returns | 3,392 | 4,528 | 5,760 | 7,096 | 8,546 |
| Closing Balance | 43,392 | 57,920 | 73,679 | 90,775 | 109,321 |

## The longevity risk and investment risk at the individual level

Just as for investment returns, longevity risk in Defined Contribution funds operates at an individual level. For the purposes of this paper, this risk is defined as the probability that the individual will live to a lesser or greater age than the population median for his/her gender.

However, the utility gained or lost by the individual is likely to be asymmetric dependant on whether the individual lives older than the mean age, or less than the mean age of their cohort. For the individual who lives less than the mean age there are several arguments (based on the ‘overlapping generations’ and ‘bequest’ models) that the individual gains utility by bequeathing the balance of their funds to their progeny.

For those that live more than the mean age, the individual gains utility in living, but loses utility in the bequest motive and in their own consumption. (In Australia this is partially offset by access to the ‘age-pension’)

The various components in constructing longevity risk at an individual level are shown in **Table 3** below:

Table 3 Effect on the individual of longevity

| **Outcome** / **Utility** | Individual lives less than the median age of cohort | Individual lives more than median age of cohort |
|---|---|---|
| **Gains** | • Greater utility from being able to pass on greater amount to the next generation | • Greater utility from a greater life span<br>• Greater utility from access to safety net |
| **Losses** | • Reduced utility from a shorter lifespan | • Reduced utility as a result of running out of money |

**Figure 3** below is a chart of expectations of life by age of survival. A survivor at 65 is in fact likely to achieve a greater age at death than is likely for one at birth, merely by virtue of surviving to age 65. (The graphs based on the Australian Life Tables[5]).

[5] Australian Bureau of Statistics: ABS cat. no. 3302.0.55.001 Life tables, Australia, 2003–2005 and author's calculations

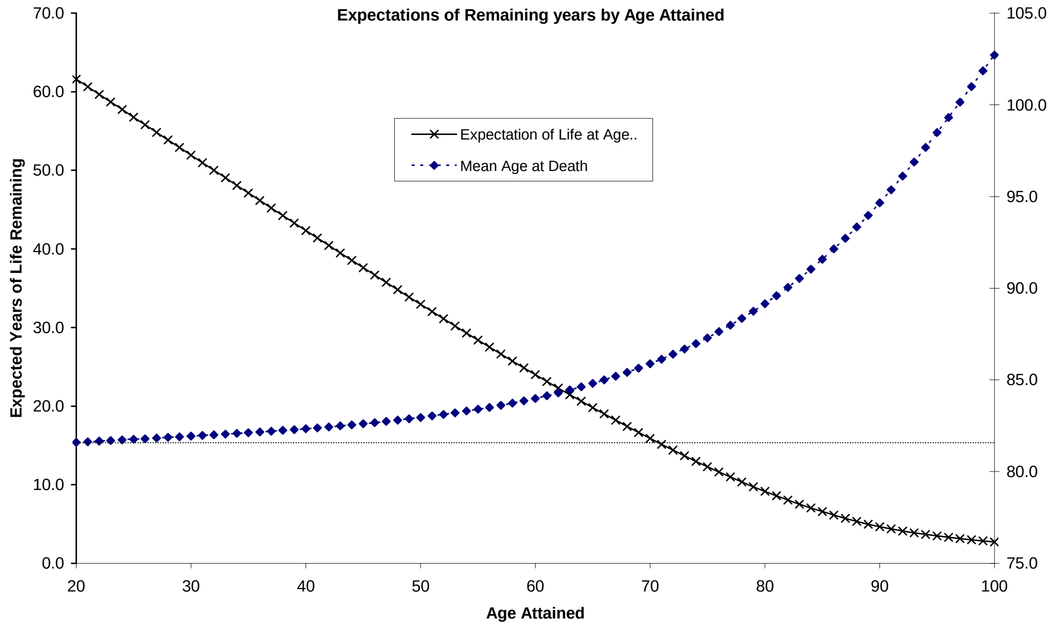


Figure 3 Australian expected years of life remaining versus age attained

The tables show that individuals expected age at death increases very slowly up until middle age, and increases more rapidly after age 65. This creates an interesting dilemma! Longevity risk is reasonably stable until the investor retires, after which, as the investor ages, their chance of living longer increases. As a result, survivors, by virtue of their survival alone, are subject to greater and greater personal longevity risk. This may have implications for survivors' risk profiles, as we shall see later.

Mitchell (2006) also showed that investors are unreliable predictors of the impact of time on the value of money. Furthermore, Livanas (2006B), using choice modelling, showed that investors did not respond to differing time horizons with changes in favoured investment risk-return profile, thereby not optimising on the basis of time. Furthermore, investors' assessment of risk was based on immediately historical experience, either in their individual returns or of the market.[6]

Constructing a function for an individual in the face of longevity risk requires the individual to maximise utility in the face of consumption versus bequest; the risk of money running out versus social security (age pension) top ups; capacity to work; risk aversion and social security top ups.

Nishiyama's model may be relevant given that it models investor reaction to partially privatised retirement savings system – which seems to correspond to the practice in Australia and the USA. Reviewing the household value function as developed by Nishiyama (2005) we see:

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[6] Confidential industry research also shows that investors will review their portfolio, on average around once a year up until retirement. From then onwards they will review their portfolio on average twice a year. Furthermore, investors will use these returns as signals for their risk assessment, and while they may not respond to change funds or investments immediately (largely because of inertia), they will nevertheless have a change in their perceptions of their utility, with losses providing a greater disutility per unit than gains (Livanas 2006C).

$$v(s_i, S_t; \Psi_t) = \max u_i\,(c_i, h_i) + \beta\,(1+\mu)^{\alpha\,(1-\gamma)}\,\Phi_i\,E\,[\,v\,(s_{t+1}, S_{t+1}\,;\Psi_{t+1}\mid e_i\,]$$

Nishiyama's value function includes survivorship ($\Phi_i$), and coefficient of risk ($\gamma$), and time preference $\beta$, $s_i$ –the state of the household – as a function of age; hourly earnings; starting wealth and historical earnings; as well as the usual references to consumption c, capacity to work (captured by the utility function) and the state of the economy $S_i$ and $\Psi$ – a given policy schedule for social security and retirement funding among others

The utility function is defined as:

$$u(c_t, h_t) = \frac{\left\{\left[\left(1+\frac{n_t}{2}\right)^{-\zeta} c_t\right]^{\alpha} \left(h_t^{max} - h_t\right)^{1-\alpha}\right\}}{1-\gamma}$$

where:

$n_i$ = the number of dependant children;

$\zeta$ is the adult equivalency scale,

$h^{max}$ is the maximum working hours;

$h_i$ is the hours worked.

Following on from the fact that survivorship is largely stable under 65, it is also the pattern that people over 65 will have fewer and fewer dependants. Nishiyama's model is interesting in that it provides a specific savings response to variations in social security, in household age and by wealth (earning ability) octiles. This provides a more granular approach to many other models. However Nishiyama seems to include a static risk factor for all age cohorts and furthermore, doesn't allow for response to variation in investment returns for common wealth cohorts.

It is precisely this dispersion of returns that forms an important factor in assessing household response and may result in changes in risk attitude varying by age. A household in reality is being asked to consider a joint probability function of longevity risk and investment returns and adjust investment accordingly.

If we exclude the capacity of the individual to defer retirement or to limit consumption, we see that the individual need to maximise utility in the face of a joint probability of:{longevity risk: return risk}, with survivorship by its nature adding longevity risk, while immediate past underperformance, reducing risk propensity.

The longevity risk is the risk (in years) of living past the mean age for the individual's cohort; and return risk as the risk (in percentage ) of achieving a return less than the mean for the asset risk class can be described by a value function at age is, as follows:

$$v(i) = P(E_{ox} > \text{mean}) \cdot P(\text{Return} < \text{mean})$$

Where $E_{ox}$ is the individuals expected age at death.

The longevity risk is then:

$$LR = \Pi\ (1-q_x)$$

Where: $q_x$ is the proportion of people dying at exactly age x.

The longevity risk is shown graphically in figure 4 for survivors at specific ages, and is based on the Australian Life Tables and author's calculations weighted for males and females together:

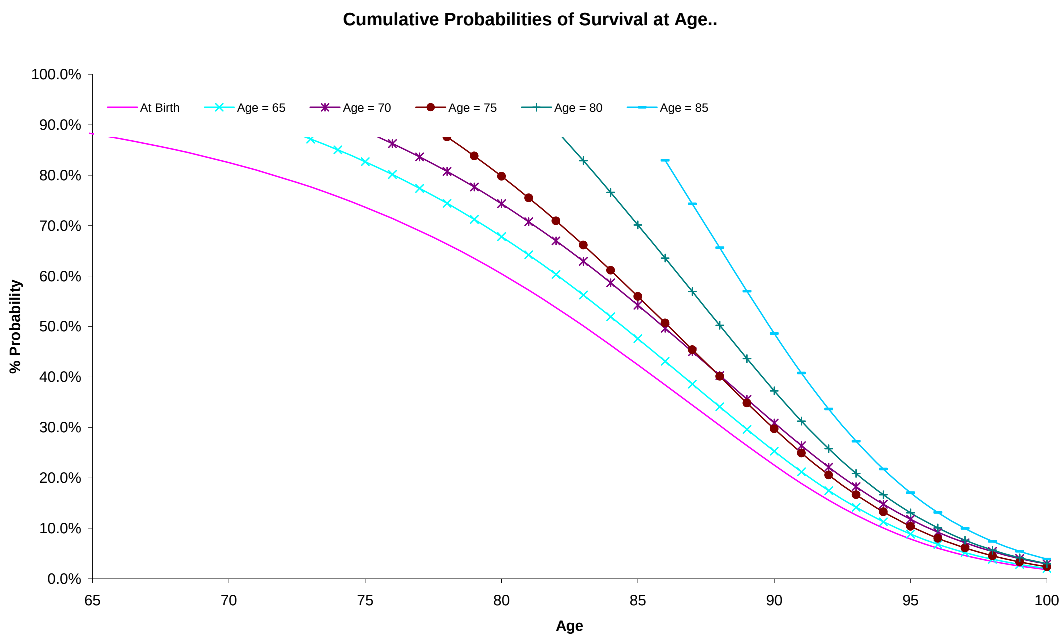



Figure 4 Cumulative probabilities of survival at birth; ages 65 to 85 (in 5 year cohorts)

If, in turn, we assume that the probability of achieving a return less than the mean in any one year is assumed Gaussian, and the returns of an efficient market over any time horizon with a positive risk free rate, are lognormal such that:

$$E = Be^{rt}$$

Where: E = End Value, B = Beginning Value and r is the return rate

Then commonly, the probability of achieving a target return value (TRV) over a time horizon with a number of periods n is given by:

$$Z_{TRV} = \frac{Log_e\left(\frac{T}{B}\right) - Log_e(1 + R_g)n}{S\sqrt{n}}$$

Where: *B* = Beginning Value

*T* = Target Value

$R_g$ = Geometric Average of holding period returns

*S* = Standard Deviation of the logs of (1+ Holding Period Returns)

So then, the joint effects of the longevity risk, and the probability of not achieving a targeted future value can be defined as:

(Probability of living longer than mean) *and* (Probability of achieving an average return below the expected mean for the risk taken)

An individual therefore is likely to assess the likelihood of running out of money as a result of investment returns in conjunction with the probability of living to the expected age (83). It is this joint assessment that this paper is addressing.

## Modelling the effects of Longevity Risk and Target Return Risk

A simulation model (Appendix 2) was set up to assess the likely distribution of savings for someone who is retiring at 65 and expecting to live to 83 (the gender weighted mean age at death as estimated from the Australian Life Tables).

The model was calibrated to assume the following parameters:

Table 4 Model Parameters

| Parameter | Value | Explanation |
|---|---|---|
| Age at retirement = | 65 | |
| Expected Age at Death = | 83 | |
| Amount at Retirement = | $200,000 | |
| Bequest at age 83 | Nil | |
| Expected Return | 9.1% | Annual Return[1] |
| Expected Standard Deviation | 5.2% | Annual Standard Deviation[2] |
| Consumption Inflation | 3% p.a. | |

Annualised median return for 5 years ending April 2007 for the default investment option of funds reporting to SuperRatings

Annualised standard deviation for 5 years ending April 2007 for the default investment option of funds reporting to SuperRatings

The model was first run to establish a baseline of consumption (or portfolio drawdown) for which there would be no terminal value of funds at death. The baseline assumed that returns would be paid at the end of each year after consumption had occurred, that the expected returns would equal the annual median return of 9.1% without variation and that the individual would be retiring at 65, with $200,000 in assets.

The model was solved for a Year 1 consumption figure that would achieve all these objectives given a further assumption that consumption would be expected to increase each year by 3%.

Once the baseline model had been calibrated, the model was run 100 times, with the only variation being the investment return each year. Each run allowed the annual investment return to vary in a Monte Carlo style simulation, according to the mean and standard deviation of the default funds as defined in **Table 4**. A further check was put in place to ensure that the average of the returns and the average of the standard deviations of all 100 runs were similar to that of the baseline model. When these were within 1 basis point of the expected returns, the model was ‘locked down’.

The purpose of the modelling was to determine impact on the individual rather than to assess the aggregate. Too often models miss the impact on individuals; it is the individual behavioural response that this paper is interested in. While the aggregate of the mean and standard deviation of the returns is known, the individual response will be dependent on the individual experience, and with asymmetric responses to losses versus gains, the sum of the individual responses will be different from the imputed response of the aggregate result.

The specific focus of the model is for retirees over 65. The model can be run for other age groups, and greater complexity can be added to allow for the interaction of the potential for additional earnings, lowered consumption, deferred retirement, and the costs of children. Nishiyama's model perhaps can be modified to achieve this, but for the purposes of assessing longevity risk, the model assumes survivors at 65.

The simulations are contained (in some detail) in Appendix 3.

The pattern of people achieving an individual return which allows then achieve their objective of funding their consumption until death at 83,  is shown in Figure 5 below:

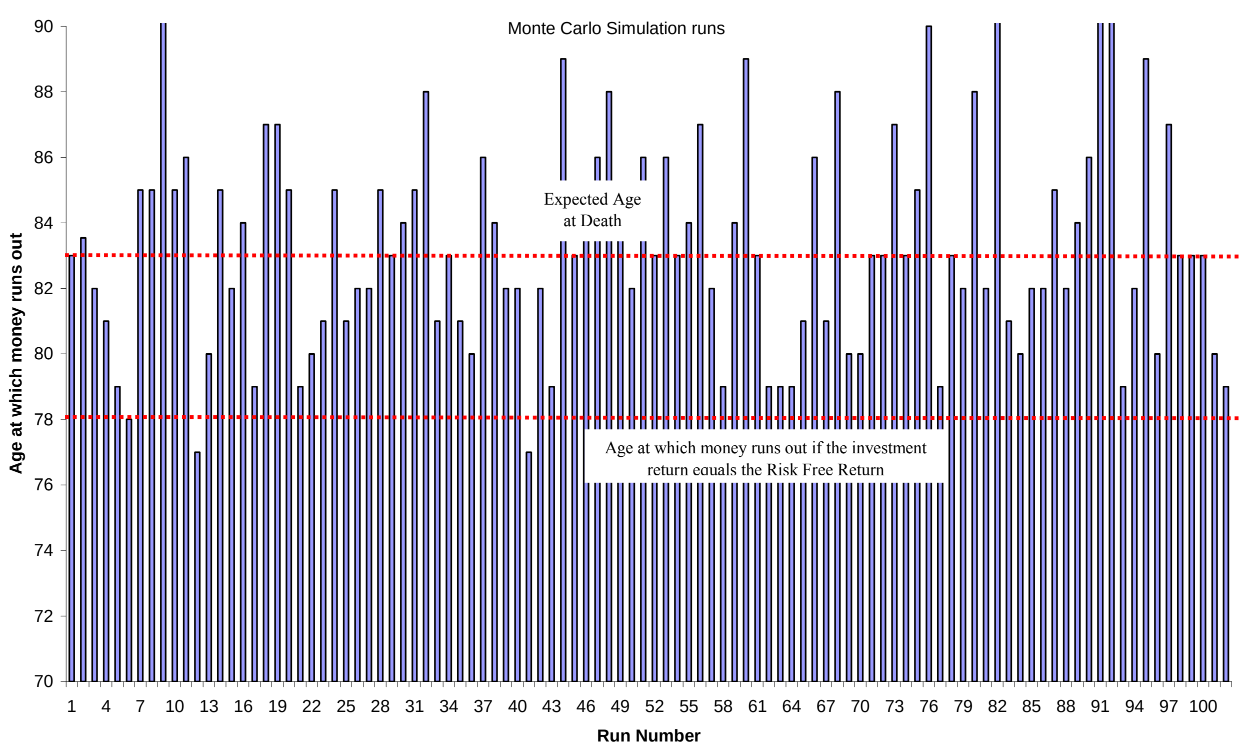
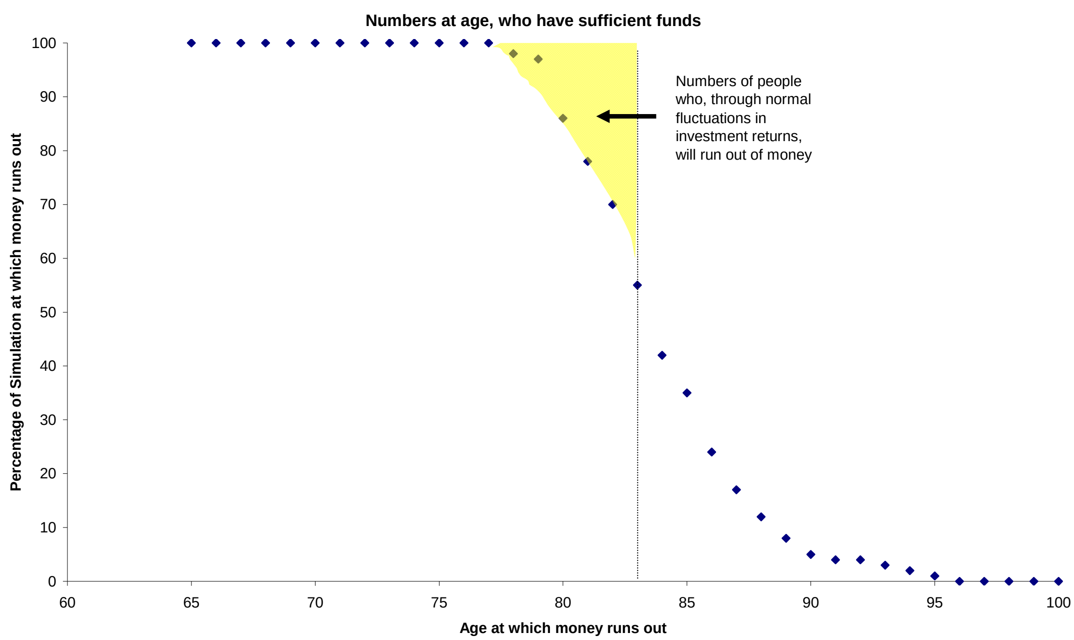


Figure 5 Chart of Runs for individuals given constant consumption patterns

Unsurprisingly, a large number of people will run out of money prior to death – assuming again that all die at exactly 83. This is due to the natural variation in returns and their effect on the individual's portfolio as modelled in Appendix 3. Nevertheless, the alternative of individuals investing in the risk free rate (4.70%, or the cash rate of return from the superannuation funds reporting to SuperRatings), will result in 100% of individuals running out of money at age 78![7] As a result, a policy response to limit investment risk is not efficient.

Figure 6 below develops Figure 5, and shows a plot of the distribution of the results by age at which the money runs out. It continues the story that, while longevity risk is important, the additional risk through normal variation in investment returns is often overlooked.




[7] This model is available on request and is merely the baseline model run at a rate of return of 4.70%.

Figure 6 Plot of numbers from Runs, at age which money runs out

By age 80, with an expectation of life until age 83, close to 15% (according to the simulation) will have run out of money as a result of natural variation. These people will be faced with an equivalent risk to those expecting to live three years longer than 83.

Given that this simulation is only modelled on the current standard deviation and returns from the aggregate of default funds, it is likely that a greater variation in returns will be experienced if the entire universe of investment options was selected. Given then that the model is concerned asymmetrically with returns (i.e. underachieving targeted returns), it is conceivable that the impact will be greater than shown in figure 6. This is developed further in the next section.

Drawing together the theme of investment risk and longevity risk, **Figure 7** below shows the probabilities of each on a single chart:

The chart shows that, on an individual level, people will face one of four scenarios – not surviving to anticipated 'average' age, and either having enough money to live to this 'abbreviated age', or through a variation in returns, even then running out of money. Alternatively they can face longevity risk, and run out of money, or, through a variation in returns, actually have money to continue their previous consumption patterns.

In **Figure 7**, a survivor at age 65 has a probability of 40% of surviving as additional 3 years.

Of course the corollary also exists – and the figure below shows that investment returns will, through natural variation, provide sufficient funds for around 15% of people who survive until age 86.

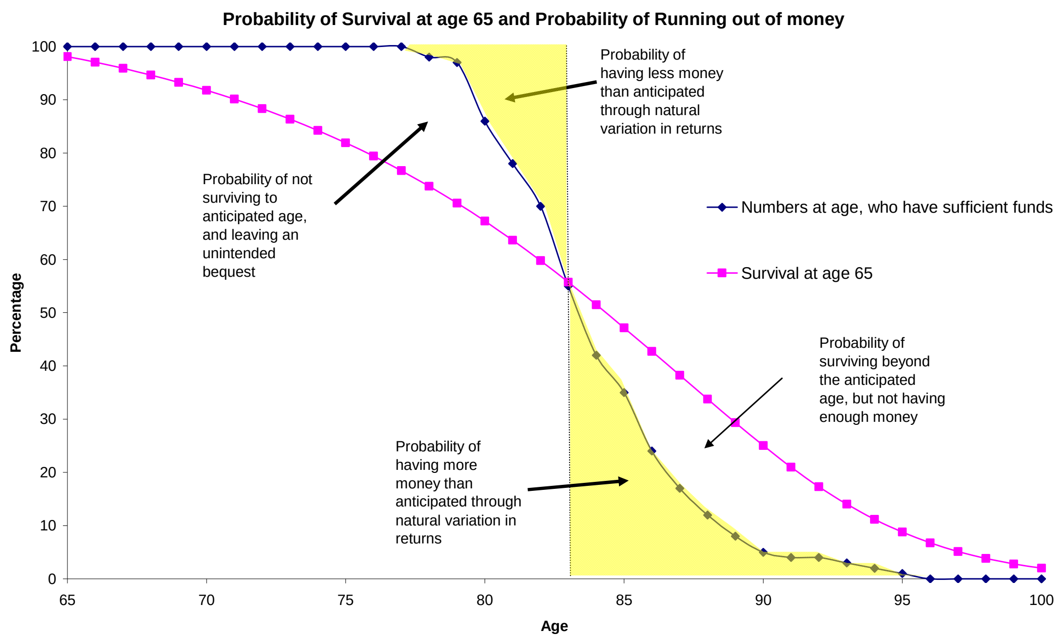



Figure 7 Probability of Survival and Probability of running out of money, by age

This brings two specific issues in question:

1. From a behavioural perspective, given that investors' response to losses is greater per unit loss than it is to gains, as investors age their longevity risk increases. For the number of investors who experience a temporary underperformance in their portfolio returns, these investors will tend to naturally reduce their risk profiles. Overall this may reduce average returns and potentially create an environment where the sum of Defined Contribution funds will return less than a grouped plan.

2. If the natural variations in returns at individual level create a bias in reduction of risk through an asymmetric response of investors to losses compared to gains, this should create a natural arbitrage for portfolio and longevity risk insurance, and that would add efficiency at a grouped level allowing for greater risk taking at an individual and aggregate level.

# Discussion

## The way people choose

The previous section showed that investors are subject to variation in returns from normal portfolio variations in addition to longevity risk. In response to both risks, the investor is faced with a decision to optimise their portfolio for a 'target date' that is well in the future, and then progressively review their portfolio as they become likely to live beyond the mean age for their cohort.

Furthermore, during their investment period (during both the accumulation and draw down phases), investors will gain or lose in their perception of utility merely through the act of reviewing the performance of their portfolio. This can sometimes lead to investors revising their portfolios or changing funds altogether. (Livanas 2007).

There is a common expectation that as investors approach retirement age the risk taken by their portfolio should reduce. As a result, many organisations are introducing 'life-cycle' style funds that provide for stepped reduction in portfolio risk. Bodie (2006), following Samuelson (1952) argues that investor risk profiles should not be redressed as a function of time to retirement, but only as a function of the investor's risk profile. And yet, as lifecycle style investing is becoming more popular, an emerging argument seems to be that investor risk profiles in fact do change as they near retirement or some target date, with the argument put forward that investors' capacity to recover from losses reduces. Optimising portfolio design over long periods of time is still a matter of significant controversy, and constructing portfolios that reflect investor risk accurately, still an art. It is instructive to recall Merton (1974) pp68:

> *"Unfortunately, as has been pointed out repeatedly, the mean-variance criterion is rigorously consistent with the general expected-utility approach only in the rather special cases of a quadratic utility function or of Gaussian distributions on security prices – both involving dubious implications……….recent dynamic simulations have shown that the behaviour over time of some efficient mean-variance portfolios can be quite unreasonable.."*

Further, Merton pp73 argues that optimising utility:

> *"..will not involve portfolio decisions constant through time.."*

This section will question whether investor utilities do indeed change as the risks that they face change.

## Behavioural Response to Risk and Return

Figure 7 above and the Australian Actuarial tables show that the longevity risk is largely the same until around age 65, at around which point survivorship starts increasing the chance of the individual living longer than the mean for their cohort.

Figure 7 also imputes that the variations in the return of a portfolio constructed with a target date in mind, will result in differences in the perspective of an individual.

Both these factors operate only at an individual level. At an aggregate level, the mean of the portfolio returns will prevail and the expected return will, in all likelihood, be achieved.

However, if the asymmetric responses of individuals to losses and gains (Kahneman 1979 and Livanas 2006B) are compiled, the overall "risk" profile of the aggregate of all portfolios is likely to be reduced.

To demonstrate simply:

$\sum EU_i$ (Gains) + $\sum EU_i$ (Losses) < $\sum EU_t$ (Net of Gains and Losses)

Where $EU_i$ is the Expected Utility for the ith investor, $EU_t$ is the aggregate expected utility of the universe. (This can be set in relative terms at zero)

Livanas (2007) showed that investors are seemingly unable to contemplate accurately differences in portfolio time horizons where they are greater than a year. So therefore, a rational response to this, is that for those investors who see that their portfolio returns are less than anticipated as they near their 'target date ' (or age at which they expect to die), they will reduce risk given that they have experienced a relative loss. Investors therefore will rely on the immediate history in their behavioural response, rather than in a forward looking assessment of time remaining for their portfolio.

This also explains people's response to longevity risk insofar as the risk of living longer than their cohort should in theory require greater risk taking in order to have a chance to meet their objectives of 'not running out of money'. People's inability to accurately contemplate future returns (to understand the time value of money or indeed time horizons in portfolios) means that investors will respond to immediately prior stimuli.

On an industry wide level, a rational response is to reduce the likelihood of loss by offering a lower risk portfolio given that the utility of losses at an individual level outweigh the utility of gains, even if on an aggregate level, the response is to reduce overall risk and possibly return.

In both cases this response will reduce returns at the aggregate level. An efficiently functioning market should serve to arbitrage on the differences in utility for losses and gains to allow for greater individual risk to be taken, and thereby to allow for a greater aggregate return.

## Summary and Implications

This paper was written to consider possible behavioural impacts of longevity at an individual level. The paper reviewed the longevity risk faced by individuals, based on the Australian Life Tables. The paper then considered the investments risks faced by an individual, assuming that they were invested in the 'default' strategy[8]. Without considering other savings or the social security safety net of the 'age pension' in Australia, the paper presented results of a Monte Carlo style simulation, run for an investor who retired at age 65 and had an expectation of life until age 83. 100 runs of the model were presented and the results demonstrated that an investor who had invested in risky assets faced, in addition to longevity risk, the risk of their money running out prior to their expected death - merely as a result of natural variations in investment returns.

The model also showed that, as a corollary, the investor who lived beyond the mean age of their cohort could also, in some cases benefit from the beneficial effects of investment returns. Consequently, an investor investing in risky assets faced one of four scenarios:

- Not living to the mean age of their cohorts, but still running out of money
- Not living to the mean age of their cohorts, but leaving an unintended bequest
- Living beyond the mean age of their cohorts and running our of money (the normally stated longevity risk)

[8] Default strategies are more common in the accumulation phase. However, for the purposes of this paper, it is assumed that the investor continues to invest in the nominated default. The purpose of the paper is to illustrate the effects on individuals and their resulting behaviour, of normal variations in returns, rather than comment on aggregate strategies. Further analysis of the actual investment strategies post retirement may be interesting.

- Living beyond the mean age of their cohorts, but having sufficient money.

The paper postulated that the response of individuals facing fluctuations in returns (even as a result of natural volatility) or the uncertainty of age of death would be to become more risk averse. This is as a result of an asymmetric utility function, where losses "loom larger than gains". As such, investors would be more likely to respond to losses than gains, and the sum of the individual responses would result in more risk aversion in aggregate, than would be the expected result when looking at the mean of returns or of longevity risk.

As a result the industry is likely to respond to this behaviour of people as they age, with investment options that are increasingly conservative and have a lower volatility (and potential return). However, this response would actually be counterproductive with the result that the overall investment returns decreasing as lower risk is taken[9] and retirement funding at the aggregate level reduced.

A better response would have been for the markets to develop pooled risk products to arbitrage the natural differential between the individual pain, and group risk. Such insurance would add utility to individuals by allowing them to take greater risks through more aggressive portfolios, providing for greater overall returns and a reduction in longevity and investment risk, even after the cost of insurance. The fact that only one annuity product is on offer in Australia, and that this product provides for a very costly option implies that the inherent risk taken by individuals far exceeds that which the market is prepared to bear and calls into question whether a government response is required.

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[9] This presumes of course that higher risks will, over time equal higher returns. This papers study of 5-year returns shows that this is still a reasonable assumption.

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# Appendix 1: SuperRatings Survey of 'Default' Funds

**SuperRatings Fund Crediting Rate Survey - Default Options**

| Fund Investment Option | 5 Year Return | 10 Year Return | 5 Year Std Deviation | 5 Year Sharpe Ratio |
|---|---|---|---|---|
| Top Quartile | 10.3 | 10.1 | 4.7 | 1.1 |
| Bottom Quartile | 8.5 | 8.9 | 5.6 | 0.6 |
| All Fund Median | 9.1 | 9.3 | 5.2 | 0.9 |
| | | | | |
| Maximum | 13.7 | 11.1 | 7.1 | 1.7 |
| Minimum | 6.4 | 7.3 | 3.0 | -1.1 |

The details of the funds utilised in this survey are available commercially from SuperRatings

# Appendix 2: Baseline Model and Assumptions

| | |
|---|---|
| Assumptions: | |
| 1. Age at Retirement | 65 |
| 2. Achieving the fund mean return of | 9.10% |
| 3. Bequest (Terminal Value at Death) | $0 |
| 4. Expected Age at death of | 83 |
| 5. No capacity to rely on age pension or additional income | |
| 6. Terminal value at age 83 is | $0 |
| 7. Consumption inflation of | 3% |
| 8. Annualised Standard Deviation of | 5.20% |
| 9. Required Consumption to achieve bequest motive | -$16,819 |
| 10. Assume at age 65, Opening Balance of $200,000, assume no bequest motive; no reliance on Age Pension and no reliance on additional income | |

| Mean Return | Year 1<br>Age = 65 | Year 2<br>Age = 66 | Year 3<br>Age = 67 | Year 4<br>Age = 68 | Year 5<br>Age = 69 | Year 6<br>Age = 70 | Year 7<br>Age = 71 | Year 8<br>Age = 72 | Year 9<br>Age = 73 | Year 10<br>Age = 74 |
|---|---|---|---|---|---|---|---|---|---|---|
| Opening Balance | $ 200,000 | $ 199,850 | $ 199,136 | $ 197,790 | $ 195,737 | $ 192,896 | $ 189,177 | $ 184,482 | $ 178,701 | $ 171,718 |
| Consumption | -$ 16,819 | -$ 17,324 | -$ 17,844 | -$ 18,379 | -$ 18,930 | -$ 19,498 | -$ 20,083 | -$ 20,686 | -$ 21,306 | -$ 21,945 |
| Investment Return | $ 16,669 | $ 16,610 | $ 16,498 | $ 16,326 | $ 16,089 | $ 15,779 | $ 15,388 | $ 14,905 | $ 14,323 | $ 13,629 |
| Closing Balance | $ 199,850 | $ 199,136 | $ 197,790 | $ 195,737 | $ 192,896 | $ 189,177 | $ 184,482 | $ 178,701 | $ 171,718 | $ 163,402 |
| Annual Investment Return | 9.10% | 9.10% | 9.10% | 9.10% | 9.10% | 9.10% | 9.10% | 9.10% | 9.10% | 9.10% |

| Mean Return | Year 11<br>Age = 75 | Year 12<br>Age = 76 | Year 13<br>Age = 77 | Year 14<br>Age = 78 | Year 15<br>Age = 79 | Year 16<br>Age = 80 | Year 17<br>Age = 81 | Year 18<br>Age = 82 | Year 19<br>Age = 83 |
|---|---|---|---|---|---|---|---|---|---|
| Opening Balance | $ 163,402 | $ 153,610 | $ 142,188 | $ 128,965 | $ 113,753 | $ 96,349 | $ 76,528 | $ 54,046 | $ 28,634 |
| Consumption | -$ 22,604 | -$ 23,282 | -$ 23,980 | -$ 24,700 | -$ 25,441 | -$ 26,204 | -$ 26,990 | -$ 27,800 | -$ 28,634 |
| Investment Return | $ 12,813 | $ 11,860 | $ 10,757 | $ 9,488 | $ 8,036 | $ 6,383 | $ 4,508 | $ 2,388 | $ 0 |
| Closing Balance | $ 153,610 | $ 142,188 | $ 128,965 | $ 113,753 | $ 96,349 | $ 76,528 | $ 54,046 | $ 28,634 | $ 0 |
| Annual Investment Return | 9.10% | 9.10% | 9.10% | 9.10% | 9.10% | 9.10% | 9.10% | 9.10% | 9.10% |

# Appendix 3: Results of 100 Runs using a Monte Carlo simulation approach for a person aged 65, retired and exposed to variations in investment return

| | Baseline | AVERAGE OF ALL RUNS | Run Number | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 9.17% | 8.94% | 9.02% | 8.04% | 9.80% | 8.22% | 10.21% | 8.60% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | -$66,355 | -$71,258 | -$171,856 | -$209,830 | $44,602 | $48,653 | $228,353 | $43,397 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 3.99% | 5.45% | 5.74% | 5.40% | 4.35% | 5.06% | 5.66% | 5.63% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 82 | 81 | 79 | 78 | 85 | 85 | 93 | 85 |

| | Baseline | AVERAGE OF ALL RUNS | Run Number | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 10 | 11 | 12 | 13 | 14 | 15 | **16** | 17 |
| 1. Age at Retirement | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achievin**g the** fund mean return of | 9.10% | 9.09% | 10.28% | 8.56% | 8.11% | 9.84% | 8.53% | 8.05% | 6.98% | 9.20% |
| 3. Expected Beque**st** (Terminal Value at Death) | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| **4.** Expected Age at death of | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on ag**e** pension or additional income | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual **Terminal** value at age 83 is | $0 | -$9,209 | $56,152 | -$220,864 | -$112,876 | $45,519 | -$62,154 | $10,128 | -$140,296 | $92,019 |
| **7.** Consumption inflation of | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualis**ed Sta**ndard Deviation of | 5.20% | 5.19% | 5.18% | 6.19% | 4.64% | 4.47% | 5.74% | 6.08% | 4.83% | 6.17% |
| 9. Required Consumption to **achieve b**equest motive | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 **Age a**t which money runs out | 83 | **83.5** | 86 | 77 | 80 | 85 | 82 | 84 | 79 | 87 |

| | Baseline | AVERAGE OF ALL RUNS | Run Number | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 10.10% | 8.79% | 8.28% | 7.78% | 9.07% | 8.84% | 8.24% | 9.47% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | $99,806 | $57,924 | -$159,555 | -$105,413 | -$69,601 | $48,847 | -$90,031 | -$53,115 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 4.26% | 5.53% | 5.64% | 4.75% | 4.42% | 4.63% | 5.23% | 4.86% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 87 | 85 | 79 | 80 | 81 | 85 | 81 | 82 |

| | Baseline | AVERAGE OF ALL RUNS | Run Number | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 8.64% | 10.67% | 10.08% | 9.60% | 9.76% | 9.76% | 8.38% | 8.76% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | -$35,499 | $30,070 | -$9,038 | $19,241 | $36,847 | $115,777 | -$95,551 | -$2,730 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 3.56% | 4.45% | 5.10% | 4.52% | 6.21% | 4.82% | 4.95% | 5.43% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 82 | 85 | 83 | 84 | 85 | 88 | 81 | 83 |

| | **Baseline** | **AVERAGE OF ALL RUNS** | **Run Number** | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 8.38% | 7.75% | 9.64% | 10.30% | 10.07% | 9.29% | 7.12% | 10.61% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | -$73,063 | -$90,509 | $61,098 | $12,887 | -$58,085 | -$50,995 | -$214,638 | -$63,739 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 6.32% | 5.06% | 5.71% | 5.51% | 5.29% | 5.14% | 4.38% | 6.05% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 81 | 80 | 86 | 84 | 82 | 82 | 77 | 82 |

| | **Baseline** | **AVERAGE OF ALL RUNS** | **Run Number** | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 8.95% | 9.27% | 9.18% | 10.38% | 9.59% | 9.33% | 8.28% | 8.69% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | -$147,570 | $138,370 | -$26,564 | $39,283 | $74,198 | $131,102 | $26,356 | -$63,610 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 5.07% | 4.62% | 4.59% | 4.73% | 5.49% | 6.21% | 5.13% | 4.36% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 79 | 89 | 83 | 85 | 86 | 88 | 84 | 82 |

| | Baseline | AVERAGE OF ALL RUNS | Run Number | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 9.42% | 9.08% | 9.11% | 8.60% | 8.27% | 9.99% | 8.79% | 10.21% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | $78,794 | -$16,154 | $70,241 | -$4,998 | $6,908 | $90,810 | -$35,439 | -$166,149 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 4.82% | 4.91% | 4.04% | 4.59% | 5.32% | 5.00% | 6.10% | 6.63% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 86 | 83 | 86 | 83 | 84 | 87 | 82 | 79 |

| | Baseline | AVERAGE OF ALL RUNS | Run Number | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 9.62% | 9.98% | 8.37% | 8.73% | 8.83% | 8.10% | 7.91% | 10.10% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | $9,511 | $142,279 | -$5,836 | -$160,557 | -$141,891 | -$172,768 | -$87,206 | $75,924 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 4.98% | 4.12% | 5.36% | 5.30% | 4.04% | 4.89% | 3.95% | 4.76% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 84 | 89 | 83 | 79 | 79 | 79 | 81 | 86 |

| | Baseline | AVERAGE OF ALL RUNS | Run Number | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 8.70% | 8.97% | 9.94% | 9.10% | 9.57% | 9.72% | 8.74% | 9.11% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | -$68,361 | $132,192 | -$114,771 | -$109,623 | -$22,925 | -$10,381 | $92,028 | -$8,772 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 5.01% | 5.40% | 4.57% | 5.04% | 5.16% | 4.73% | 5.23% | 6.11% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 81 | 88 | 80 | 80 | 83 | 83 | 87 | 83 |

| | Baseline | AVERAGE OF ALL RUNS | Run Number | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 9.16% | 8.60% | 8.63% | 9.07% | 9.47% | 10.21% | 8.44% | 10.45% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | $37,058 | $177,355 | -$139,439 | -$25,858 | -$42,309 | $106,654 | -$37,861 | $211,065 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 6.30% | 4.83% | 6.12% | 5.84% | 5.99% | 4.95% | 6.37% | 5.40% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 85 | 90 | 79 | 83 | 82 | 88 | 82 | 94 |

| | Baseline | AVERAGE OF ALL RUNS | Run Number | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 8.32% | 8.76% | 7.91% | 8.43% | 9.67% | 8.21% | 8.85% | 9.51% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | -$84,415 | -$112,349 | -$53,384 | -$39,530 | $43,438 | -$30,540 | $5,305 | $84,193 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 3.48% | 5.64% | 4.74% | 5.17% | 4.06% | 4.60% | 5.54% | 6.25% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 81 | 80 | 82 | 82 | 85 | 82 | 84 | 86 |

| | Baseline | AVERAGE OF ALL RUNS | Run Number | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 10.40% | 10.07% | 9.16% | 9.88% | 9.91% | 9.62% | 9.69% | 8.63% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | $187,147 | $273,590 | -$181,761 | -$58,483 | $152,482 | -$139,754 | $97,362 | -$8,479 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 4.96% | 5.33% | 6.01% | 5.31% | 5.82% | 5.70% | 4.69% | 5.70% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 92 | 95 | 79 | 82 | 89 | 80 | 87 | 83 |

| | Baseline | AVERAGE OF ALL RUNS | Run Number | | | |
|---|---|---|---|---|---|---|
| | | **1** | 98 | 99 | 100 | 101 |
| 1. Age at Retirement | 65 | **65** | 65 | 65 | 65 | 65 |
| 2. Achieving the fund mean return of | 9.10% | **9.09%** | 8.75% | 8.96% | 7.44% | 8.51% |
| 3. Expected Bequest (Terminal Value at Death) | $0 | **$0** | $0 | $0 | $0 | $0 |
| 4. Expected Age at death of | 83 | **83** | 83 | 83 | 83 | 83 |
| 5. No capacity to rely on age pension or additional income | | **0** | 0 | 0 | 0 | 0 |
| 6. Actual Terminal value at age 83 is | $0 | **-$9,209** | -$8,761 | -$13,804 | -$108,271 | -$148,111 |
| 7. Consumption inflation of | 3% | **3%** | 3% | 3% | 3% | 3% |
| 8. Annualised Standard Deviation of | 5.20% | **5.19%** | 4.84% | 5.01% | 6.09% | 5.11% |
| 9. Required Consumption to achieve bequest motive | -$16,819 | **-$16,819** | -$16,819 | -$16,819 | -$16,819 | -$16,819 |
| 10 Age at which money runs out | 83 | **83.5** | 83 | 83 | 80 | 79 |

# Appendix 4: Summary of results of the Monte Carlo Simulation

| Age | | Numbers at age, who have sufficient funds |
| --- | --- | --- |
| Age less than | 100 | 0 |
| Age less than | 99 | 0 |
| Age less than | 98 | 0 |
| Age less than | 97 | 0 |
| Age less than | 96 | 0 |
| Age less than | 95 | 1 |
| Age less than | 94 | 2 |
| Age less than | 93 | 3 |
| Age less than | 92 | 4 |
| Age less than | 91 | 4 |
| Age less than | 90 | 5 |
| Age less than | 89 | 8 |
| Age less than | 88 | 12 |
| Age less than | 87 | 17 |
| Age less than | 86 | 24 |
| Age less than | 85 | 35 |
| Age less than | 84 | 42 |
| Age less than | 83 | 55 |
| Age less than | 82 | 70 |
| Age less than | 81 | 78 |
| Age less than | 80 | 86 |
| Age less than | 79 | 97 |
| Age less than | 78 | 98 |
| Age less than | 77 | 100 |
| Age less than | 76 | 100 |
| Age less than | 75 | 100 |
| Age less than | 74 | 100 |
| Age less than | 73 | 100 |
| Age less than | 72 | 100 |
| Age less than | 71 | 100 |
| Age less than | 70 | 100 |
| Age less than | 69 | 100 |
| Age less than | 68 | 100 |
| Age less than | 67 | 100 |
| Age less than | 66 | 100 |
| Age less than | 65 | 100 |