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Applying the 6-factor Nelson-Siegel Model to Mortality Rate Curves

Weiyu Kuo
Department of International Business
National Chengchi University

Chenghsien Tsai (corresponding author)
Associate Professor, Department of Risk Management and Insurance
National Chengchi University
No. 64, Section 2, Chihnan Road, Taipei, Taiwan, 11623
(Tel) +886-2-2936-9647; (Fax) +886-2-2939-3864; E-mail: ctsai@nccu.edu.tw

Ko-Lun Kung
Ph.D. Student, Department of Risk Management and Insurance
National Chengchi University

Hsiao-Tzu Huang
Post Doctor, Department of Risk Management and Insurance
National Chengchi University

Abstract

Lee and Carter (1992) developed an extrapolative method for modeling and forecasting mortality, but this model is known to be over-parameterized. We propose an alternative method to overcome the over-parameterization problem. We borrow the concept in Nelson and Siegel (1987) and describe the force of mortality by an exponential function. The exact model that we adopt is the 6-factor Nelson-Siegel model extended by Rezende (2008). The two parameters of the model, λ_1 and λ_2, are determined by an optimization algorithm called the particle swarm optimization (PSO). Given λ_1 and λ_2, the OLS estimate of β_i of the 6-factor model can be computed easily. Our tested data include the mortality rates of females and males in 12 countries. The preliminary results show that most in-sample fittings of the 6-factor Nelson-Siegel model are better than the benchmark Lee-Carter model, except in case of one France.

Key words: mortality models; Lee-Carter model; Nelson-Siegel model

1. Introduction

Lee and Carter (1992) developed an extrapolative method for modeling and forecasting mortality. Both the academia and the industry quickly noticed this method. Many extended versions and applications of the Lee-Carter model have been developed and adopted. The initial concept of the Lee-Carter model is based on analysis of long term trends, and it is employed to forecast US mortality. Currently, the Lee-Carter model seems to be a standard for fitting and forecasting mortality.

However, using the Lee-Carter model can be difficult or even problematic when the data is massive or comparisons are made between different countries. The difficulties are caused by the over-parameterization problem.

This study attempts to search a model to overcome the over-parameterization problem.

We are inspired by the interest rate literature and the recent development of stochastic mortality modeling (Cairns, Blake and Dowd, 2006; Girosi and King, 2007; and Hari, Waegenaere, Melenberg and Nijman, 2008). We found that some significant similarities exist between the features of interest rate models and those of mortality models, e.g., the instantaneous interest rate and the force of mortality. It is thus reasonable to apply the Nelson-Siegel class of yield curve model as an alternative way to depict the mortality curve. We borrow the original idea in Nelson and Siegel (1987) and describe the force of mortality by an exponential function. The exact model that we adopt is the 6-factor Nelson-Siegel model extended by Rezende (2008).

The data of mortality rates we have used for in-sample fitting comes from the Human Mortality Database[1]. The data include mortality rates of both females and males in 12 countries during the period 1950 to 2006. We compare the 24 fittings results from the 6-factor Nelson-Siegel model with those of the Lee-Carter model. Our preliminary results show that most fitting results of 6-factor Nelson-Siegel model are better than the Lee-Carter model, except in case of France.

2. Lee-Carter Model

The Lee-Carter model is a dynamic and discrete time model proposed by Lee and Carter in 1992. Lee and Carter constructed the model based on the historical data of US mortality from 1933 to 1987. They used a new model to forecast mortality at each age. Because of its easy comprehensibility and applicability for each age level, the Lee-Carter model has become very popular in the industry, as well as academia, in many nations. The US Bureau of Census has used this model to forecast long-run life expectancy (Hollmann et al., 2000). Furthermore, two Social Security Technical Advisory Panels have suggested their

[1] The data can be download from their website: http://www.mortality.org

Trustees adopt the Lee-Carter model forecasts (Lee and Miller, 2001). The G7 nations also use the model to forecast their respective mortalities (Tuljapurkar et al., 2000). The Lee-Carter model is therefore used as a benchmark for fitting and forecasting performance.

The Lee-Carter model is specified as follows:

$$\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \tag{1}$$

$m_{x,t}$: central death rate at age x in year t

a_x : averaged curve of mortality at age x

k_t : general level of mortality in year t

b_x : changing rate at age x, it is the tendency of mortality when the general level of mortality (k_t) changes

$\varepsilon_{x,t}$: error term at age x in year t

The equation of the Lee-Carter model is known to be over-parameterized (Haberman and Renshaw, 2008; Li and Chen, 2007). The over-parameterization problem results in heavy calculations and the difficulties of comparing mortality among different countries. The main purpose of this study is to find a method to simplify the mortality forecasting model with better accuracy.

3. Nelson-Siegel Family Models

Nelson and Siegel (1987) developed an interest rate model for forecasting interest rate structure in central banks. In this model, the yield rate is generated by a Laguerre function plus a constant term. The Laguerre function enables the Nelson-Siegel model to have the capability to fit various shapes of yield curve well, including a humped-shape yield curve. The Nelson-Siegel model contains three exponential components which represent the long term, short term and medium term factors for yield rate. The following parametric model is used to fit the forward curve:

$$y(x,t) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda_{1,t}x}}{\lambda_{1,t}x}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda_{1,t}x}}{\lambda_{1,t}x} - e^{-\lambda_{1,t}x}\right) \tag{2}$$

The constant λ regulates the decay rate of the whole curve. This is because the λ controls both decaying speed of the exponential component of $\beta_{2,t}$ and the maximum point of the exponential component of $\beta_{3,t}$.

Svensson (1994) proposed a four-factor SV model which increased the Nelson-Siegel model's flexibility through an additional factor. However, the SV model cannot be made no-arbitrage. This encouraged Christensen, Diebold and Rudebusch (2008) to construct a five-factor model to solve the problem. Here we decide to use the 6-factor model proposed by Rezende (2008). The main features of the 6-factor Nelson-Siegel model are the fusion of smoothness and flexibility. The model is specified as follows:

$$y(x,t) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda_{1,t}x}}{\lambda_{1,t}x}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda_{2,t}x}}{\lambda_{2,t}x}\right) + \beta_{4,t}\left(\frac{1-e^{-\lambda_{1,t}x}}{\lambda_{1,t}x} - e^{-\lambda_{1,t}x}\right) + \beta_{5,t}\left(\frac{1-e^{-\lambda_{2,t}x}}{\lambda_{2,t}x} - e^{-\lambda_{2,t}x}\right) + \beta_{6,t}\left(\frac{1-e^{-\lambda_{1,t}x}}{\lambda_{1,t}x} - e^{-2\lambda_{1,t}x}\right) \tag{3}$$

4. Data and Methodology

The mortality data are obtained from the Human Mortality Database. We select 12 countries to include in our sample. Each of them has two (male and female) groups of mortality data. Therefore, we deal with 24 data groups to fit the mortality model. The sample period is from 1950 to 2006 that is the latest mortality data available as of 2009. The mortality data we fit are the crude mortality rates of males with ages of 30 to 100.

We estimate β_i by a two-step procedure used in Nelson and Siegel (1987). The estimation process is as follows.

1. Obtain the OLS estimates of β_i through different calendar years, from a given $\lambda_{i,t}$.

2. Select $\lambda_{i,t}$ that minimizes the total sum of squared error of different calendar years.

Firstly, the parameter $\lambda_{i,t}$ is searched through the optimization algorithm, i.e., PSO. Given pre-specified decay parameter $\lambda_{i,t}$, the OLS estimate of β_i can be computed easily. We then calculate the sum of squared error (SSE) of each fitted mortality curve. To determine the value of $\lambda_{i,t}$, we sum up SSE in every year for which the mortality curve is fitted, and then choose $\lambda_{i,t}$ such that it minimizes the sum of the SSEs. It is important to note that we do not compute $\lambda_{i,t}$ for every calendar year. Instead, we select a single $\lambda_{i,t}$ that fits the best for every calendar year.

5. Preliminary Results

The Nelson-Siegel class model becomes linear once $\lambda_{i,t}$ are set. We fit the mortality rate to the 6-factor Nelson-Siegel model using standard OLS technique and obtain fitted value of β_i. In order to compare the degree of fit of the two models, we adopt Mean Absolute Percentage Error (MAPE) to measure the fit of the models.[2] Most results of the total data groups are better than the Lee-Carter model, except in case of France (see Table 1).

[Insert Table 1 Here]

[2] The equation of MAPE as: MAPE= $\frac{1}{n}\sum_{t=1}^{n}\frac{|\varepsilon_t|}{X_t}\times 100$

6. Future Work

o enhance the robustness of the evaluation, we propose to make more varied assessments with our model in the future. Furthermore, the study will proceed to comparing the forecasting capabilities of our model with those of the Lee-Carter model.

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Table 1 Results of in-sample fitting of the 6-factor Nelson-Siegel (NS) model and the Lee-Carter (LC) model during 1950-2006.

Country/Sex	Model	Avg. of MAPE	Avg. of MAPE(NS-LC)
Australia_female	NS model	0.0106	**-0.0039**
	LC model	0.0146	
Australia_male	NS model	0.0123	**-0.0068**
	LC model	0.0191	
Finland_female	NS model	0.0155	**-0.0095**
	LC model	0.025	
Finland_male	NS model	0.0181	**-0.0085**
	LC model	0.0265	
France_female	NS model	0.0157	0.0034
	LC model	0.0123	
France_male	NS model	0.0158	0.0007
	LC model	0.0152	
Japan_female	NS model	0.0073	**-0.0227**
	LC model	0.0301	
Japan_male	NS model	0.0132	**-0.0083**
	LC model	0.0215	
Netherlands_female	NS model	0.0107	**-0.0028**
	LC model	0.0135	
Netherlands_male	NS model	0.0123	**-0.0111**
	LC model	0.0234	
NewZealand_female	NS model	0.0214	**-0.0024**
	LC model	0.0238	
NewZealand_male	NS model	0.0206	**-0.005**
	LC model	0.0256	
Norway_female	NS model	0.0164	**-0.0024**
	LC model	0.0189	
Norway_male	NS model	0.0163	**-0.0043**
	LC model	0.0205	
Spain_female	NS model	0.0142	**-0.0053**
	LC model	0.0194	
Spain_male	NS model	0.0151	**-0.0047**
	LC model	0.0199	
Sweden_female	NS model	0.0135	**-0.0035**
	LC model	0.017	
Sweden_male	NS model	0.0154	**-0.0051**

	LC model	0.0205	
Switzerland_female	NS model	0.0191	**-0.0001**
	LC model	0.0192	
Switzerland_male	NS model	0.0185	**-0.0021**
	LC model	0.0207	
US_female	NS model	0.0096	**-0.0004**
	LC model	0.01	
US_male	NS model	0.0094	**-0.0033**
	LC model	0.0127	
UnitedKingdom_female	NS model	0.0093	**-0.0024**
	LC model	0.0117	
UnitedKingdom_male	NS model	0.0114	**-0.0059**
	LC model	0.0173	