

HEDGING PURE ENDOWMENTS WITH MORTALITY DERIVATIVES

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ABSTRACT. In recent years, a market of mortality derivatives are appearing as a solution for increasing systematic mortality risk inherent in insurance contracts. The systematic mortality risk is due to the uncertain development of future mortality intensities. In this paper, we develop a theory for pricing insurance company issued pure endowment when hedging with a mortality forward is allowed. The mortality intensities associated with the pure endowment and the reference mortality intensities for the mortality forward are correlated and are modeled by stochastic processes. We price the pure endowment assuming that the issuing company hedge its contract with the mortality forward and requires compensation for unhedgeable part of the mortality risk in the form of a pre-specified instantaneous Sharpe ratio. We show that the value of the pure endowment is identical to the upper good deal bound of Cochrane and Saá-Requejo [2000] and Björk and Slinko [2006] applied to our setting. The major result of this paper is that the value per contract solves a linear partial differential equation as the number of contracts approaches infinity. One can present the limiting price as an expectation under an equivalent martingale measure. Another important result is that hedging with the mortality forward may raise or lower the price of this pure endowment comparing to its price without hedging determined in Milevsky et al [2005]. The market price of reference mortality intensities and the correlation between the two portfolios jointly determine the cost of hedging. We demonstrate our results using numerical examples.

Key words. Life annuities, longevity risk, q-forward, mortality-linked derivatives, instantaneous Sharp ratio, incomplete market