Optimal Purchase of Life and Longevity Risk Insurance Products for Retired Couples

Andreas Hubener, Raimond Maurer, and Ralph Rogalla

Longevity Risk Conference, September 2011

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Motivation and Research Question

Uncertainty of lifetimes is major risk for retired couples:

- risk of outliving their assets and leaving too little bequest
- risk of losing the income of the deceased spouse

How can this risk be hedged by the dynamic allocation using

- term life insurance
- single annuities
- joint and survivor annuities
- stocks and bonds

combinations

Literature and Contributions

Brown and Poterba (2000, JRI): welfare gains of full annuitization in joint and survivor annuities for couples in retirement

Horneff et al. (2008, JRI): dynamic annuitization and portfolio choice in retirement framework

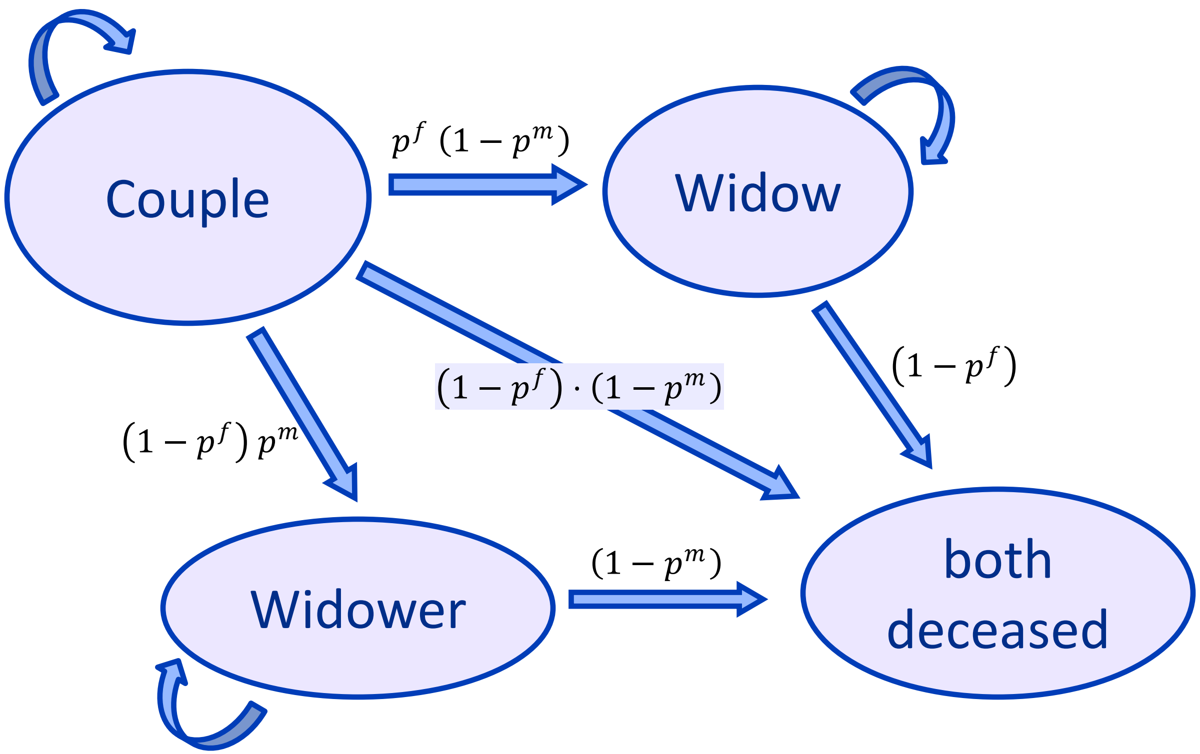
Love (2010, RFS): dynamic life cycle portfolio choice for families - investment universe: stocks, bonds, life insurance

Our contribution: discrete time portfolio choice model for a couple in retirement with dynamic annuitization and life insurance purchases

Our Model – Family State

Family State

- “family” s is Markov chain with four states:
  - couple – widow – widower – both deceased
- transitions are only mortality driven (→ no divorce etc.) and given by the individual one-year survival probabilities.



Our Model – Preferences

Family Preferences

- utility is gained from consumption and bequest in CRRA framework (RRA $\gamma = 5$; time pref. $\beta = 0.96$)

$$J_t = \max\{u(C,s) + \beta\, E_t[\,J_{t+1}]\}$$

Markov chain

- consumption is normalized by consumption scaling factor ϕ_s ("effective family size")

→ couple: $\phi_s = 1.3$ singles: $\phi_s = 1$

$$u(C,s) = \frac{1}{1-\gamma}\left(\frac{C}{\phi_s}\right)^{1-\gamma}$$

- bequest Parameter *B = 2* gives the strength of the bequest motive

$$\text{Bequest} = \frac{1}{1-\gamma}\left(\frac{W_t}{B}\right)^{1-\gamma}$$

Our Model – Financial & Insurance Products

Financial and Insurance Products

- liquid wealth can be invested in
  - riskless bonds (interest rate: $R_f - 1 = 2\%$)
  - risky stocks (risk premium 4%, volatility 15.7%)
- renewable one-year term life insurance for each spouse

$$\underbrace{LP_t}_{\textit{price}} = \underbrace{L}_{\textit{face-value}} \cdot \frac{(1-p_t)}{R_f}$$

- single annuities for each spouse

$$\underbrace{AP_t}_{\textit{price}} = \underbrace{A}_{\textit{pay-ments}} \cdot \underbrace{\sum_{\tau=t+1}^{T} \frac{p_{\tau,t}}{\left(R_f\right)^{\tau-t}}}_{\ddot{a}_t \ \textit{annuity factor}}$$

- joint annuities – constant payments till the last spouse dies

$$p_{\tau,t}^{j} = p_{\tau,t}^{f} + p_{\tau,t}^{m} - p_{\tau,t}^{f} p_{\tau,t}^{m}$$

Our Model – Financial & Insurance Products

Joint and Survivor Annuities: Survivor Benefit Ratio

- upon first death payments are reduced to survivor benefit ratio K
- Annuity pricing factor: $$\ddot{a}_t^K = (1-K)\underset{\textit{2 single}}{\left(\ddot{a}_t^f + \ddot{a}_t^m\right)} + (2K-1)\underset{\textit{joint (K=1)}}{\ddot{a}_t^j}$$
- single and joint annuities (K=1) allow for any survivor benefit ratio K
- the overall annuity holdings of the family can be seen as a combination of:
  - j&s annuity with a specific survivor benefit ratio, and
  - an additional single annuity for one spouse
- example:

model

wife	$A^f = 4$
husband	$A^m = 3$
joint (K=1)	$A^j = 4$

interpretation

j&s (K=0.7)	$A^{K=0.7} = 10$
wife	$A^f = 1$

Our Model – Policies

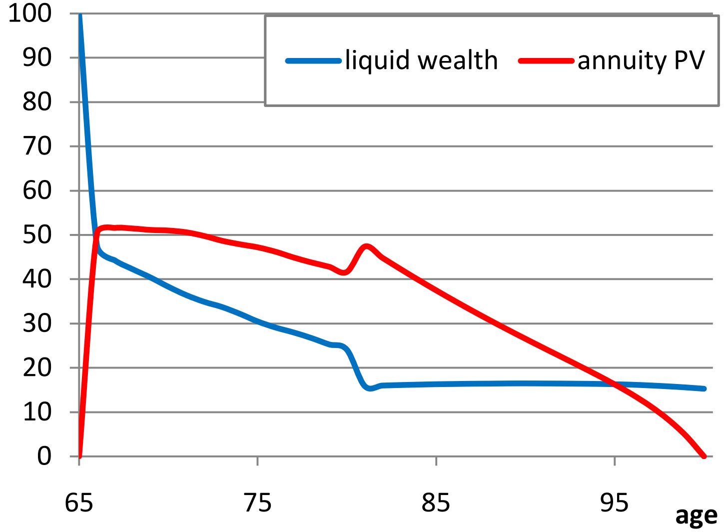
Decision Variables in each Period:

- consumption
- expenditures on life insurances
  - wife
  - husband
- expenditures on annuities
  (availability is restricted to maximum age)
  - wife
  - husband
  - joint
- allocation of (remaining) liquid wealth to stocks and bonds

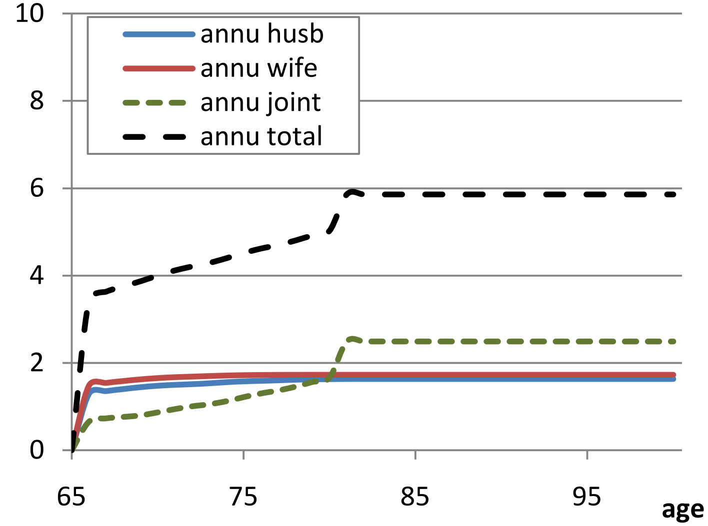
Solution for optimal decisions found by value function iteration.

Life Cycle Profile without Pre-Annuitized Wealth

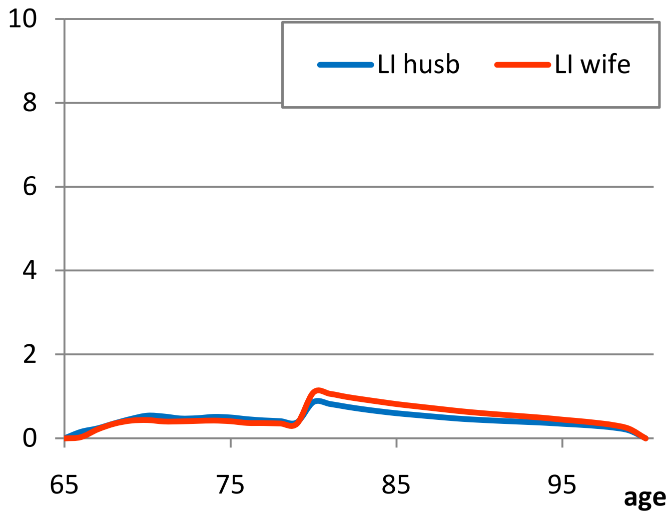
Wealth



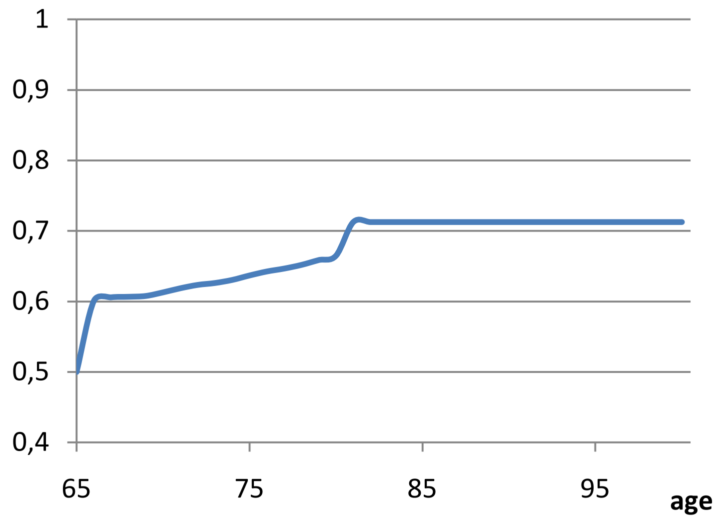
Annuity payments



Life insurance (face values)

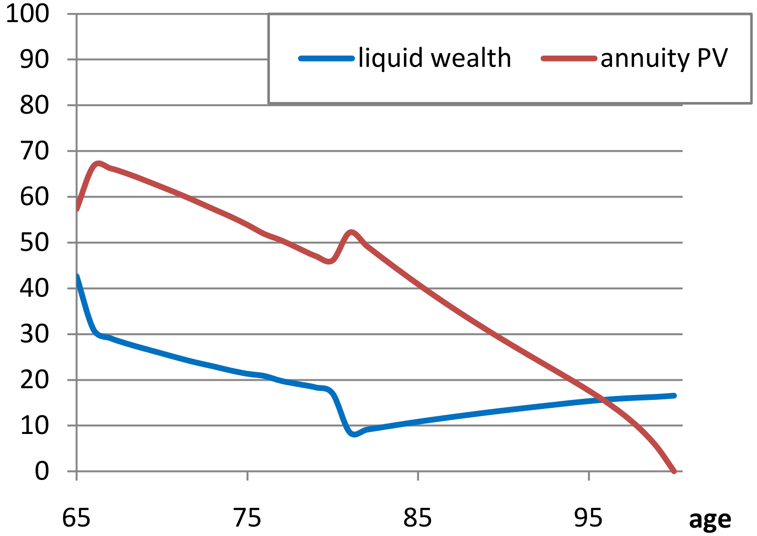


Survivor benefit ratio K

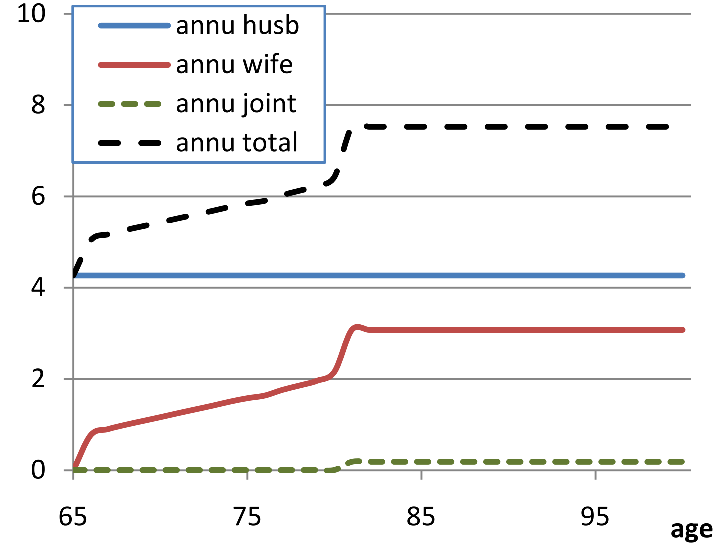


Life Cycle Profile with Pre-Annuitized Wealth (husband)

Wealth



Annuity payments



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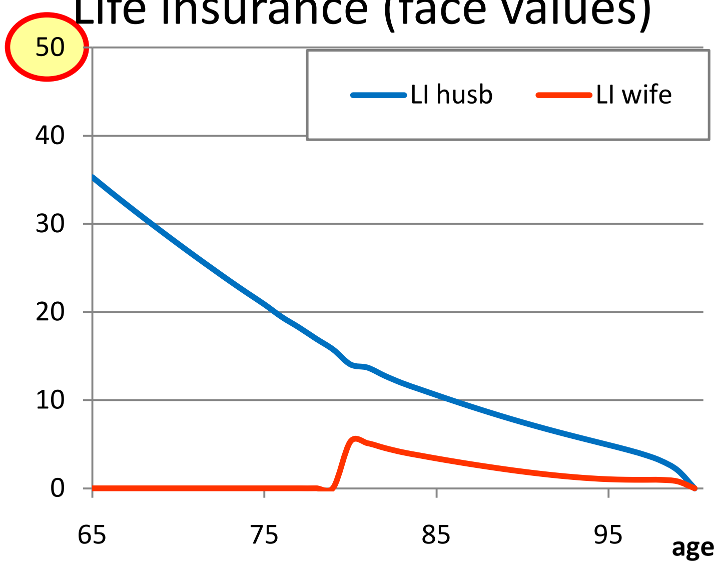
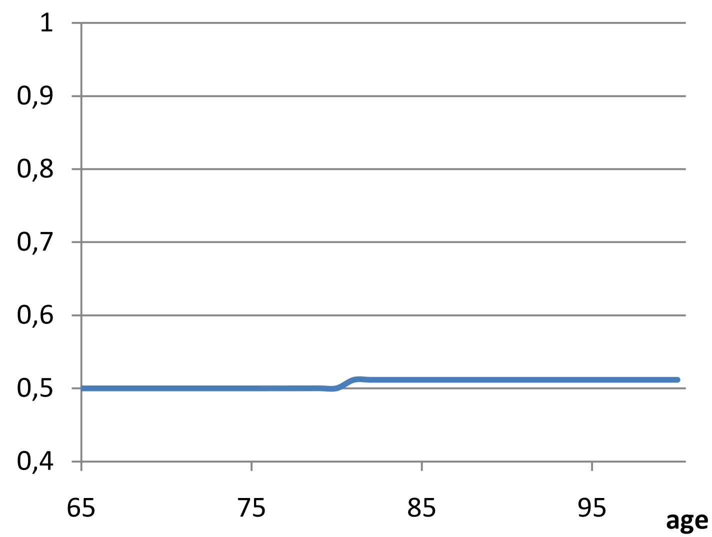


Survivor benefit ratio K



Welfare Analysis

- certainty equivalent at age 65: $$CE_{65} = \left((1-\gamma)\cdot J_{65}\right)^{\frac{1}{1-\gamma}}$$

	no pre-annuitization	with pre-annuitization (only husband)
annuities available up to **age 80**	1.00	0.98
no annuities available at all	0.83	0.86
annuities available up to **age 65**	0.99	0.94
annuities available up to **age 80** **no life insurance**	0.999	0.86

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Conclusion

- Joint & survivor annuities are useful products to hedge both kinds of mortality risk.
- Liquid wealth (invested mainly in stocks) is preferred over life insurance for bequest.
- Life insurance is used to insure pre-annuitized retirement wealth (e.g. DB pensions) of one spouse. Then they yield high welfare gains.

Thank

you !

Backup

Effect of Consumption Scaling on Survivor Benefit Ratio

- Household may purchase annuities only at the beginning of retirement
- What is the survivor benefit factor of the optimal j&s annuity?

consumption scaling factor	survivor benefit ratio K
1.00	1.000
1.10	0.924
1.20	0.848
1.30	0.784
1.40	0.729
1.50	0.681
1.60	0.639
1.70	0.602
1.80	0.570
1.90	0.540
2.00	0.516