ROBUST HEDGING OF LONGEVITY RISK

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Plan

- Intro $+$ model
- Recalibration risk – introduction
- Robustness questions – index hedging
- Are some hedging instruments more robust than others?
- Static Delta and *Nuga* hedging
- Discussion

Focus of this talk

Index-based hedges

- Customised longevity swaps only available to very large pension plans
- Index-based hedges
  - smaller schemes
  - better value for money for large plans ???
  - Quantity of hedging instrument
    Hedge effectiveness
    Price
    How confident are we in these quantities? $\Rightarrow$ ROBUSTNESS

Simple example

- Static *value* hedge: $t = 0 \longrightarrow T$
- $a_k(T, x) =$ population k annuity value at T
- Liability value $L(T) = a_2(T, 65)$
- Hedging instrument: deferred longevity swap

$$H(T) = a_k(T, x) - \hat{a}_k{}^{\mathsf{fxd}}(0, T, x)$$

$$\hat{a}_k^{\mathsf{fxd}}(0, T, x) = \text{value at } T \text{ of swap fixed leg}$$

- $k = 2$ (CMI) $\Rightarrow$ CUSTOMISED hedge
- $k = 1$ (E&W) $\Rightarrow$ INDEX hedge

Hedging: basic idea

- $L =$ liability value
- $H =$ value of hedging instrument
- Objective: minimise $Var(\text{deficit}) = Var(L + hH)$

$$\Rightarrow \text{hedge ratio, } h = -\frac{Cov(L,H)}{Var(H)} = -\rho\frac{S.D.(L)}{S.D.(H)}$$

$$\text{Hedge effectiveness} = 1 - \frac{Var(L + hH)}{Var(L)} = \rho^2$$

More general: multiple assets

$\Rightarrow$ minimise $Var(L + h_1 H_1 + \ldots + h_n H_n)$

Simple example: APC model (Cairns et al., 2011a)

$m_k(t,x) =$ population k death rate

$$\log m_k(t,x) = \beta^{(k)}(x) + \kappa^{(k)}(t) + \gamma^{(k)}(t-x)$$

$\beta^{(1)}(x),\ \ \beta^{(2)}(x)$ population 1 and 2 age effects

$\kappa^{(1)}(t),\ \ \kappa^{(2)}(t)$ period effects; mean reverting spread

$\gamma^{(1)}(c),\ \ \gamma^{(2)}(c)$ cohort effects

Key: $\nu_\kappa = \kappa^{(1)}(t),\ \kappa^{(2)}(t)$ long term trend

Realism: valuation model $\neq$ simulation model

- (Re-)calibration using data up to T $\Rightarrow$ realistic!

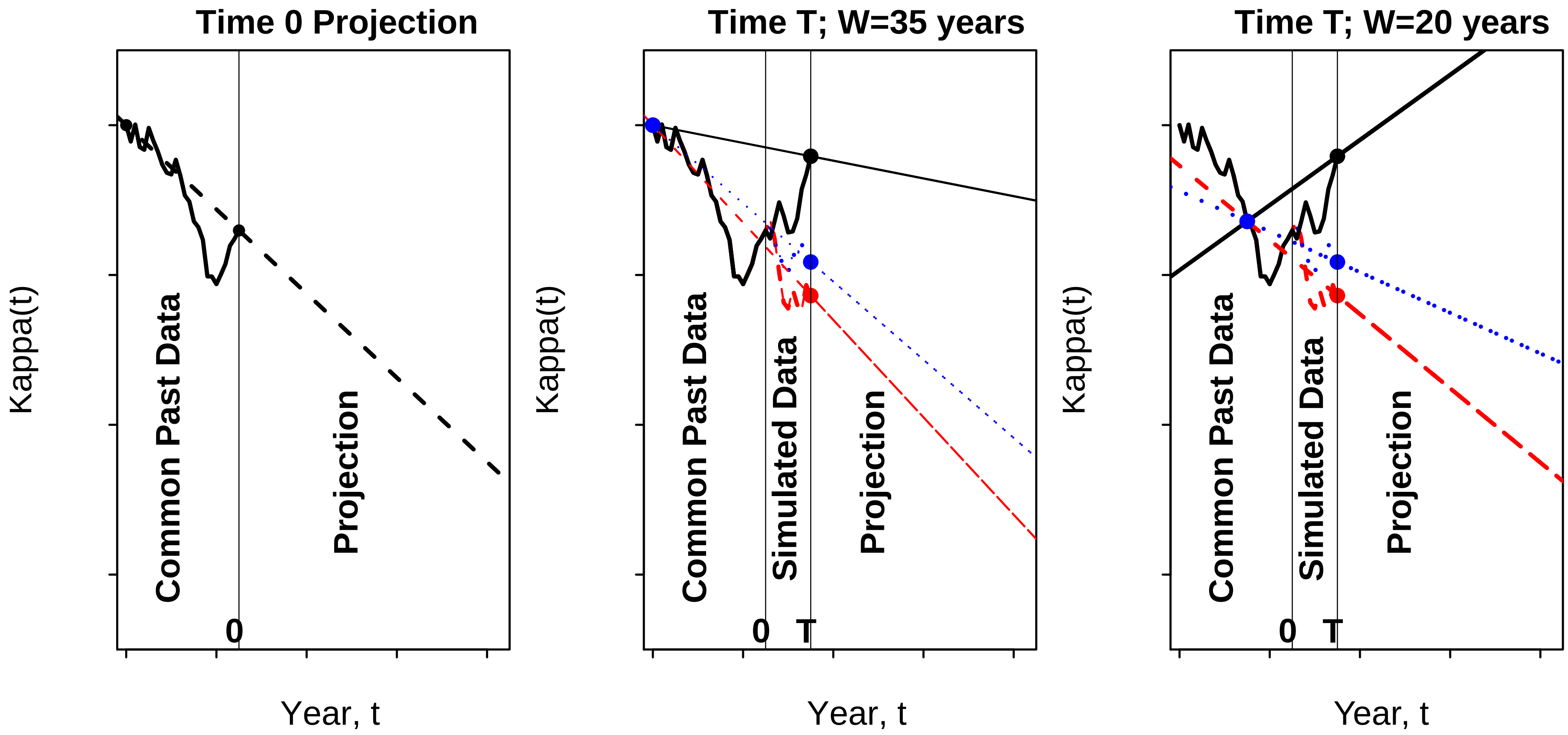
- Valuers just observe historical mortality plus

  one future sample path of mortality from 0 to T

  $\Rightarrow$ do not know the “true” simulation/true model

- Using true model $\Rightarrow$ too optimistic (??) c.f. Black-Scholes

Recalibration risk – example (random walk)



- You *will* recalibrate at T
- Recalibration depends on as yet unknown experience from 0 to T
- Recalibration depends on length of lookback window

Hedge Effectiveness: (Cairns et al., 2011b; Longevity 6)

Key conclusions: index-based hedging

- Recalibration $\Rightarrow$ risk $\nearrow$
- BUT hedge effectiveness also $\nearrow$

WHY?

Additional trend risk is common to both populations.

$$a_k(T,x) \approx f(\beta^{(k)}_{[x]}, \kappa^{(k)}_T, \gamma^{(k)}_{T-x+1}, \nu_\kappa)$$

Preliminary conclusion

Correlation and hedge effectiveness are not robust relative to the treatment of recalibration risk.

What about the hedge ratio? Price?

Robustness

How robust are estimates of:

- Optimal hedge ratios $h_1, \ldots, h_n$
- Hedge effectiveness
- Initial hedge instrument prices $\pi(H_1), \ldots, \pi(H_n)$

... relative to ...

Robustness

How robust are key quantities relative to

- Treatment of parameter risk
- Treatment of population basis risk
- Valuation model: recalibration risk (Cairns et al., L6)
- Poisson risk
- Use of latest EW data
- Simulation model $+$ calibration

Modelling Variants

- PC: Full parameter certainty (PC);

  Valuation Model NOT recalibrated in 2015

- PC-R: As full PC

  Except: Valuation Model recalibrated in 2015

- PU: Full parameter uncertainty with recalibration

- PU-Poi: Full PU with recalibration $+$ Poisson risk

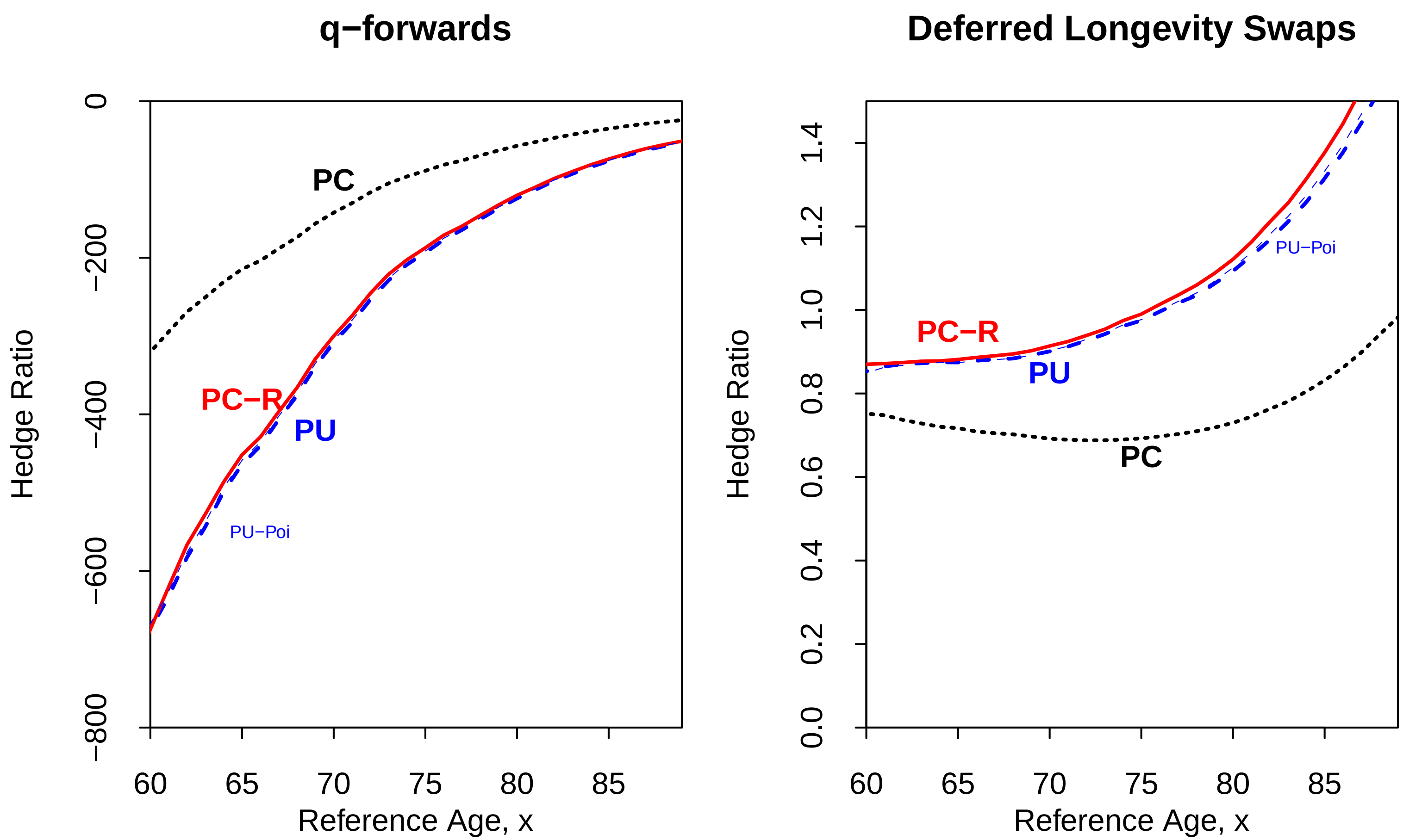
Hedging options

- Recall: Liability, $L = a_2(T, 65)$ (CMI)
- Hedging instrument (ref England & Wales):
  - $H = a_1(T, x) - a_1^{\text{fxd}}(0, T, x)$

    OR
  - q-Forward maturing at T

    $H = q(T, x) - q^F(0, T, x)$

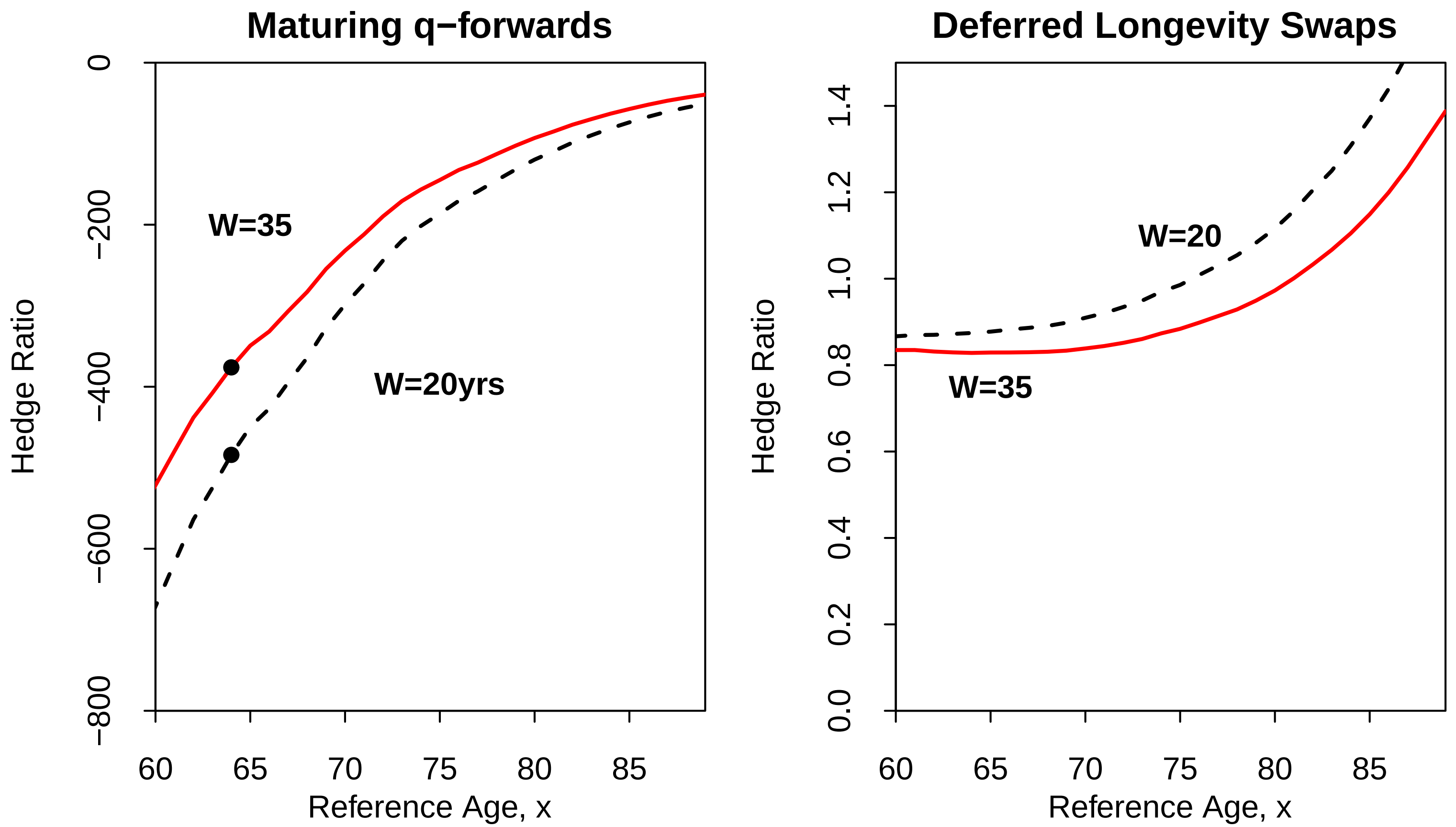
Robustness of Hedge Ratios



PC $\longrightarrow$ PC-R not robust;   PC-R $\longrightarrow$ PU robust

deferred longevity swaps better than maturing q-Forwards

Robustness relative to recalibration window, W



Deferred longevity swaps better than maturing q-Forwards

Robustness relative to recalibration window, W

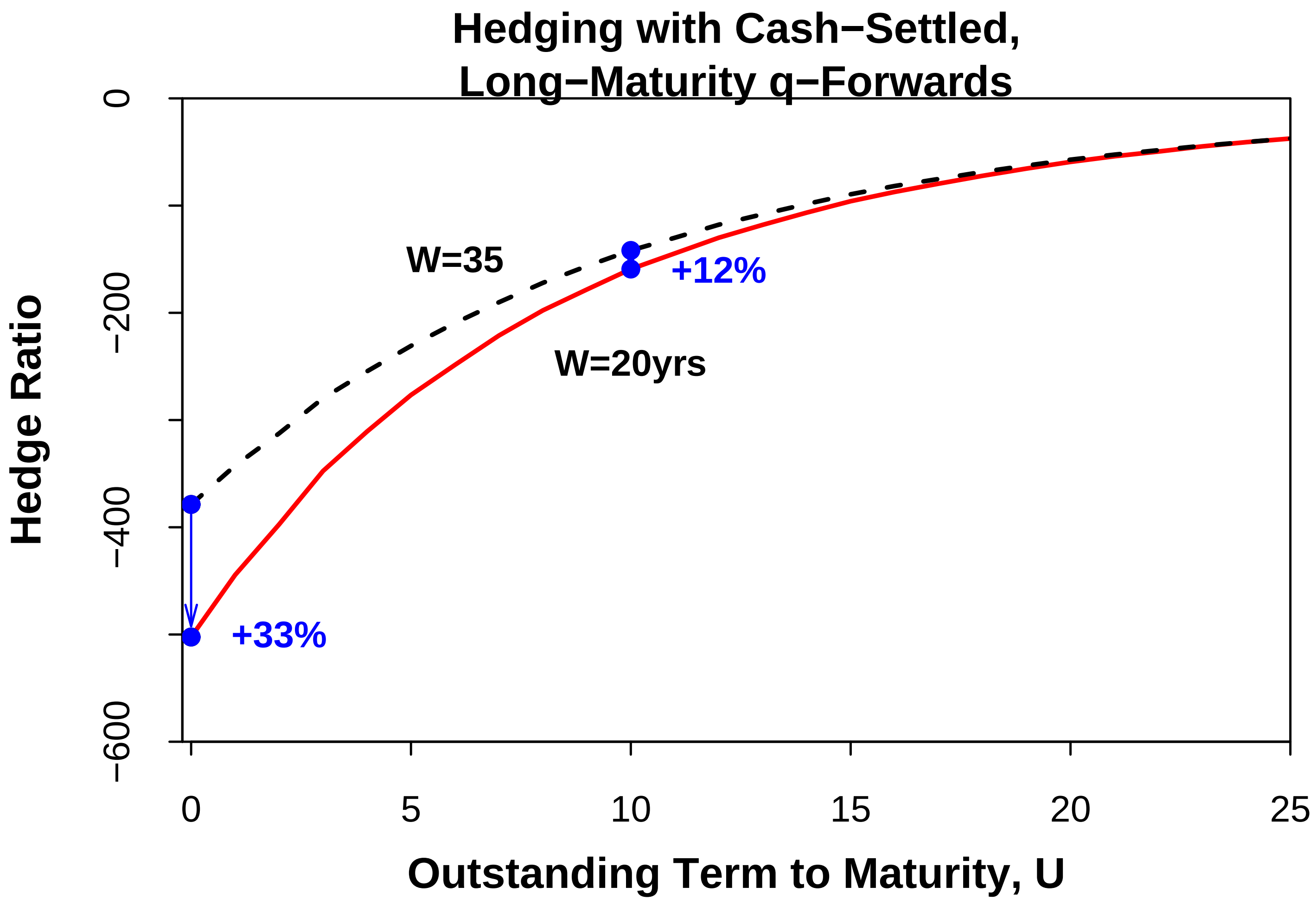
Longevity swaps are more robust:

- Liability, L, and longevity swap, H, depend on
  - $\kappa_T^{(1)}$ and ν_κ
  - BUT in differing proportions $\Rightarrow$ single H not robust
- Maturing q-Forward depends on $\kappa_T^{(1)}$ only

  $\Rightarrow$ even less robust
- Possible market solution:

  $(0, T+U, x)$ q-Forward, cash settled at T

Robustness relative to recalibration window, W



$T + U$ q-Forward is cash settled at time T

$\Rightarrow$ value depends on $\kappa_t^{(1)}$ and ν_κ

Robustness relative to recalibration window, W

- If we know W, then ν_κ linear in $\kappa_T^{(1)}$

  $\Rightarrow$ one hedging instrument sufficient

- If W is not known

  or, ν_κ determined by other methods

  $\Rightarrow$ two hedging instruments are required

  $\Rightarrow$ Delta and "Nuga" hedging

Delta and Nuga Hedging

Recall: $a_k(T,x) \approx f(\beta_{[x]}^{(k)}, \kappa_T^{(k)}, \gamma_{T-x+1}^{(k)}, \nu_\kappa)$

Liability: $L = a_2(T,x)$.

Hedge instruments: $H_1 = a_1(T,x_1) \quad \longrightarrow h_1$ units

$H_2 = a_1(T,x_2) \quad \longrightarrow h_2$ units

Delta and Nuga hedging $\Rightarrow$ require

$$\text{Deltas:} \quad \alpha \frac{\partial L}{\partial \kappa^{(2)}} = -h_1 \frac{\partial H_1}{\partial \kappa^{(1)}} - h_2 \frac{\partial H_2}{\partial \kappa^{(1)}}$$

$$\text{and Nugas:} \quad \frac{\partial L}{\partial \nu_\kappa} = -h_1 \frac{\partial H_1}{\partial \nu_\kappa} - h_2 \frac{\partial H_2}{\partial \nu_\kappa}$$

where $\alpha = Cov(\kappa_T^{(1)}, \kappa_T^{(2)})/Var(\kappa_T^{(1)})$.

Concept:

same idea as Vega hedging in equity derivatives

– hedging against changes in a parameter that is supposed to be constant.

Numerical example: $L = a_2(T, 65),\ T = 10$

$H_1 = a_1(T, 65) \quad H_2 = a_1(T, 85)$

Strategy	h_1	h_2	Var(Deficit)	Hedge Eff.	
$W = 20$					
A	0	0	0.3481	0	
B	-0.8775	0	0.03202	0.9080	(1)
C	-0.8291	0	0.03298	0.9052	(3)
D	-1.3376	0.7199	0.03209	*0.9078*	(2)
$W = 35$					
A	0	0	0.2233	0	
B	-0.8775	0	0.03353	0.8498	(3)
C	-0.8291	0	0.03289	0.8527	(1)
D	-1.3376	0.7199	0.03298	*0.8523*	(2)

Numerical example: discussion

- Annuity-Annuity hedging $\Rightarrow$ *net* Nuga-risk is modest

  $\Rightarrow$ Delta-Nuga hedging lessens the *small* gap in hedge effectiveness

- Delta-Nuga hedging will have a greater impact if
  - ν_κ subject to additional risk
  - H_1 is relatively less sensitive to ν_κ

    e.g. H_1 is a T-year q-Forward

    H_2 is a $(T+U)$-year q-Forward settled at T

	q-F$(T, 64)$	q-F$(T+T, 74)$			
Strategy	h_1	h_2	Var(Deficit)	Hedge Eff.	
$W = 20$					
A	0	0	0.3481	0	
B	500.7	0	0.03435	0.9013	(1)
C	389.0	0	0.04996	0.8565	(3)
D	-279.6	256.4	0.03797	*0.8909*	(2)
$W = 35$					
A	0	0	0.2233	0	
B	500.7	0	0.04953	0.7782	(3)
C	389.0	0	0.03392	0.8481	(1)
D	-279.6	256.4	0.03493	*0.8436*	(2)

Robustness relative to other factors

Results are robust relative to:

- inclusion of parameter uncertainty in $\beta_x^{(k)}$, $\kappa_t^{(k)}$, $\gamma_c^{(k)}$
- pension plan's own small-population Poisson risk
- index population: EW-size Poisson risk, maybe smaller
- CMI data up to 2005 $+$ EW data up to 2005

  versus

  CMI data up to 2005 $+$ EW data up to 2008

Conclusions

Robust hedging requires *inclusion* of

- Recalibration risk (Nuga)
- Careful treatment of recalibration window
- Long-dated hedging instruments to handle *Nuga* risk

Results appear to be robust relative to

- Poisson risk
- Parameter uncertainty (other than recalibration risk)
- Treatment of latest data

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