Coherent Projections of Age, Period, and Cohort Dependent Mortality Improvements

Marie-Christine Aleksic
Matthias Börger

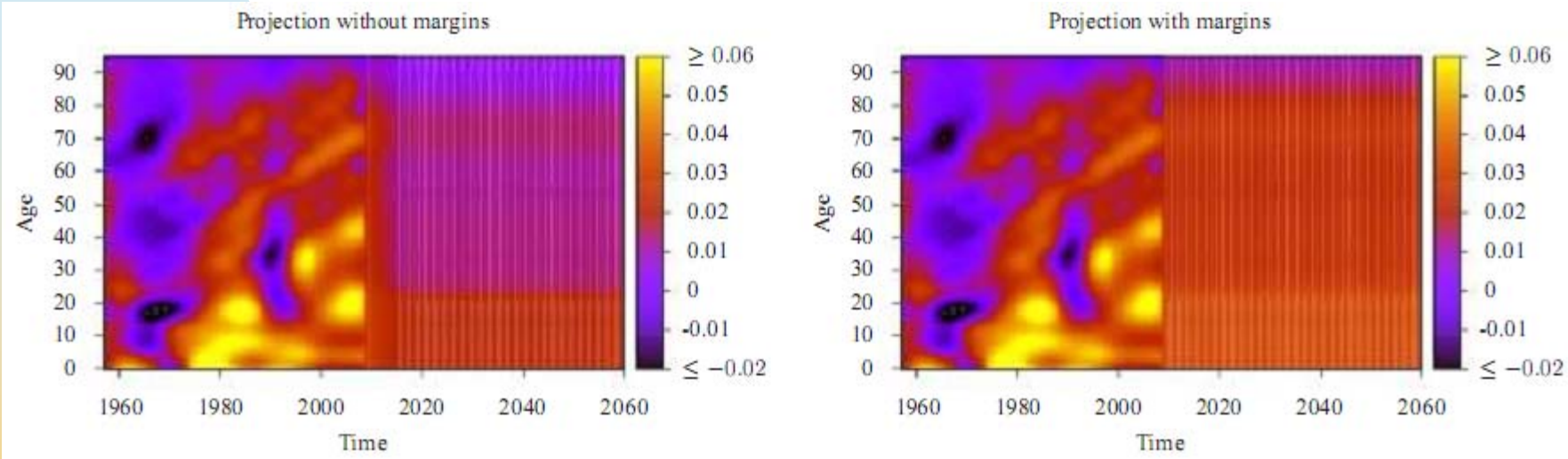
September 2011

Helmholtzstraße 22
D-89081 Ulm
phone +49 (0) 731/50-31230
fax +49 (0) 731/50-31239
email ifa@ifa-ulm.de

ifa
Institut für Finanz- und Aktuarwissenschaften

Introduction

- **Some of the currently used standard projections show significant shortcomings**
- **Example: standard projection for German annuity business**



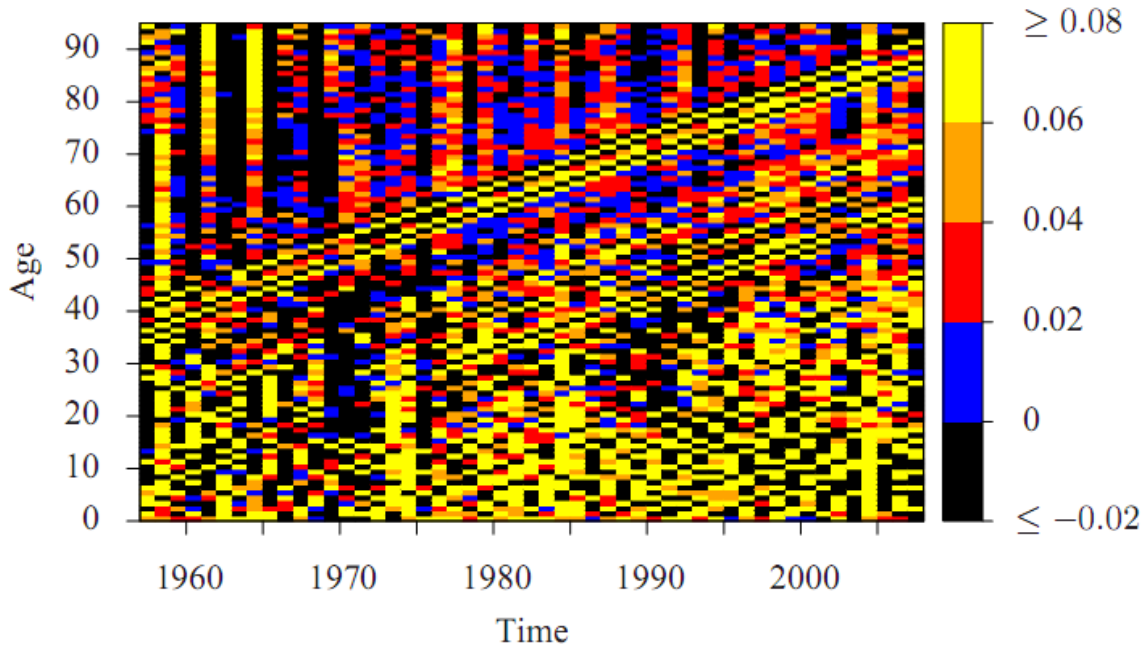
- Structural break between historical and projected improvements
- No cohort effects
- Possibly significant underestimation of future mortality improvements

→ Space for improvement of current projections

ifa
Institut für Finanz- und Aktuarwissenschaften

Model Specification

- **Raw historical mortality improvements for German males**



- **Historical data show period and cohort dependent effects**
- **Mortality improvements have often been shown to be age dependent as well**
- **We model one-year mortality improvements according to the APC model:**

$$v(x,t) = a_x + p_t + c_{t-x}$$

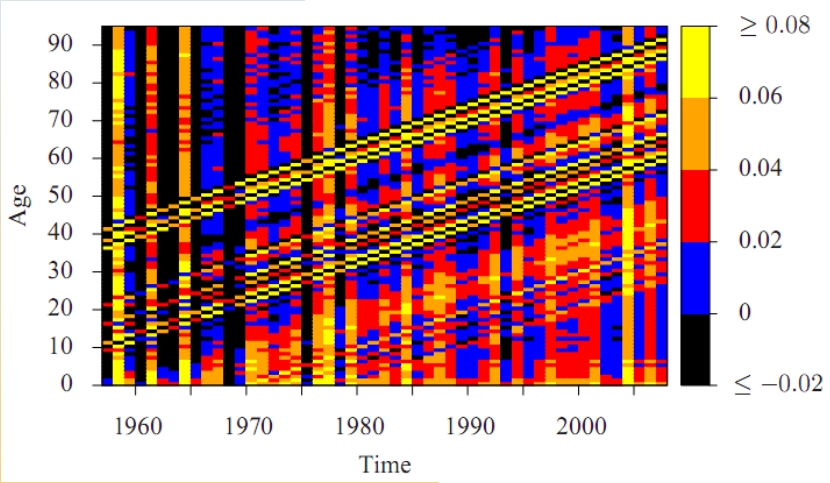
Model Constraints

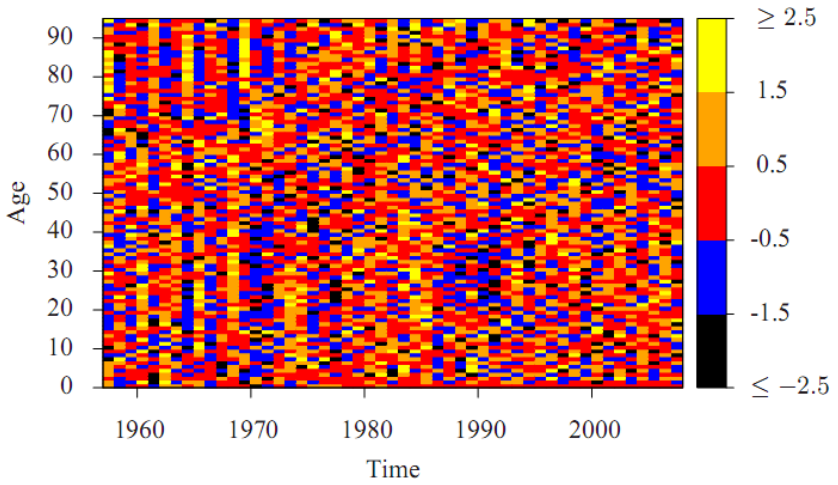
- **Random noise in cohort parameters at the boundaries**
  - Parameters are fitted to only a few data points
  - We set them to their historical average
  - Number of cohort parameters depends on the data set

- **Identifiability problem: APC model calibration is not unique**
  - Period parameters sum up to zero
  - For convenience: Cohort parameters sum up to zero
  - Thus, all “substance” is contained in the age parameters

Model Estimation

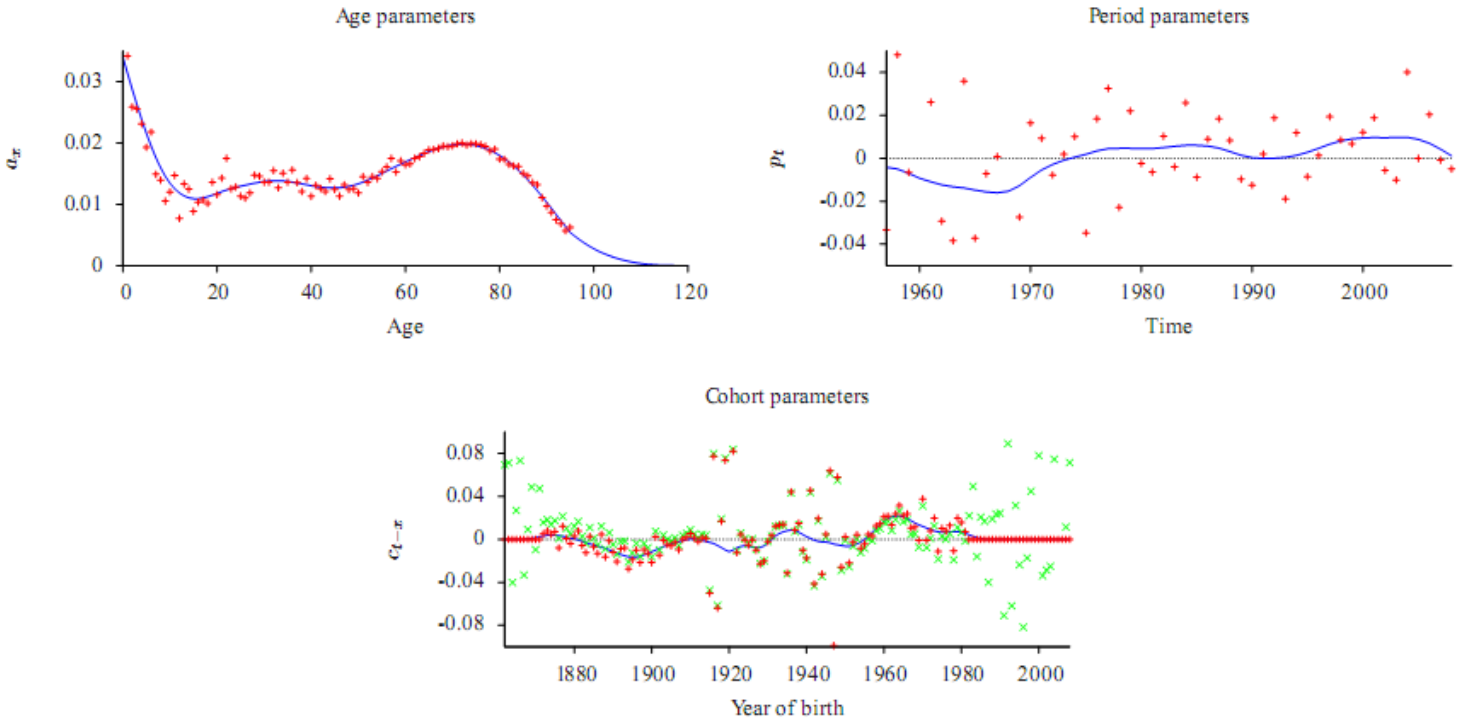
- **Model is fitted in iteratively reweighted least squares**
  - Weighting important due to stronger random fluctuations for young ages in particular
  - As weights we use empirical standard deviations from surrounding cells
  - Iteration is stopped when all model parameters change by less than 0.1%





- **Residuals for simplified model versions contain significant structure**

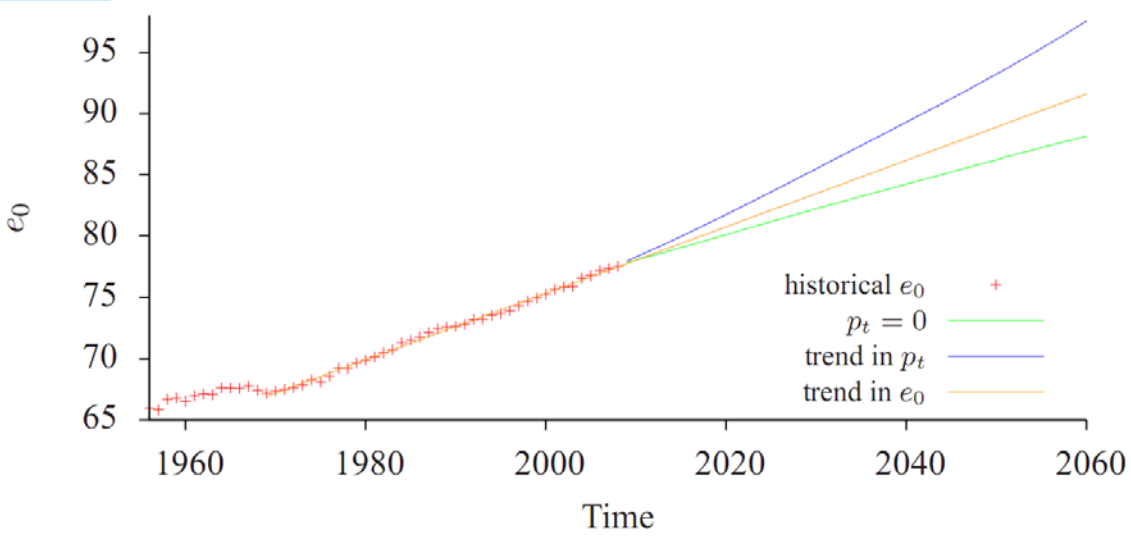
Estimation Results



- **Age parameters are extrapolated starting from age 96**
  - Extrapolations of mortality rates by different mortality laws indicate shrinking improvements
  - This observation is in line with findings of other authors, e.g. Gampe (2010)
  - We apply a cubic function which monotonically decreases to zero at age 120
  - Adjustment may be applicable to be more conservative

Projection

- **Projection uncertainty can be massive for individual countries**

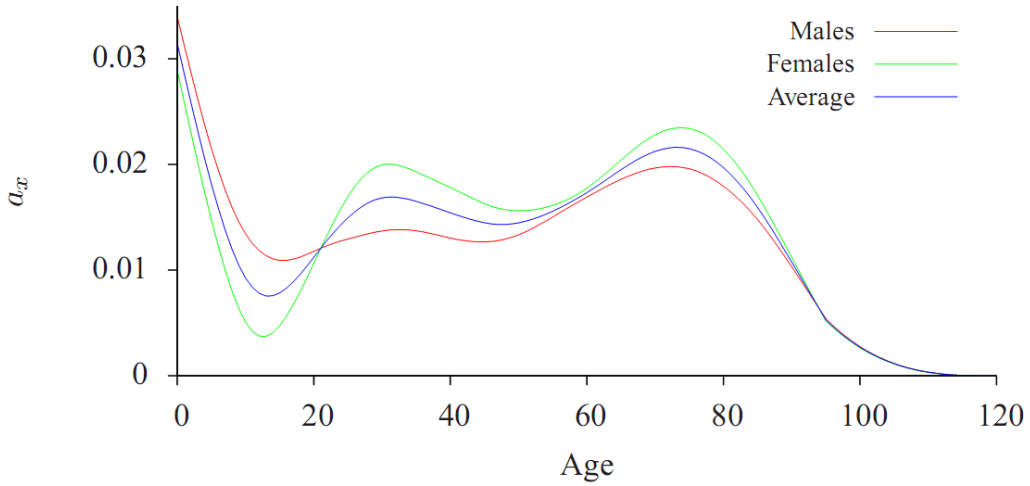


- **Information from other populations can reduce projection uncertainty**
- **Goal: Coherent Projection between**
  - Males and females in the same country (e.g. Germany)
  - Populations in different but related countries (e.g. European countries)

ifa
Institut für Finanz- und Aktuarwissenschaften

Projection of Age Parameters

- **Different age parameters between males and females lead to diverging mortality rates**
- **Thus, the age parameters should be equal in the long run**
- **Projection for both genders according to average of age parameters**
  - Appropriate if parameter values are similar for males and females



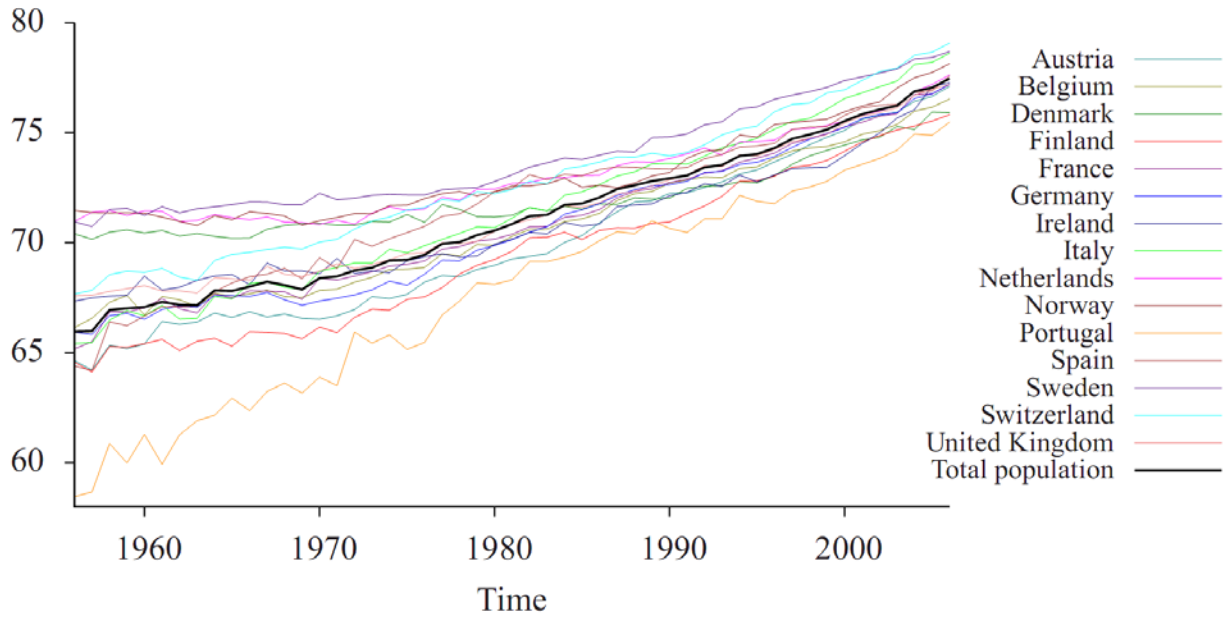
- **Analogous approach may be reasonable for age parameters for populations from different countries**

Projection of Cohort Parameters

- **Cohort effects are only temporary and thus, do not affect the long-term coherence of mortality projections**
- **Cohort effects for males and females are not necessarily correlation (cf. MacMinn and Weber (2009))**
- **We keep cohort parameters as fitted for each population individually**
- **Parameters for new cohorts are set to their long-term average of zero**

Projection of Period Parameters

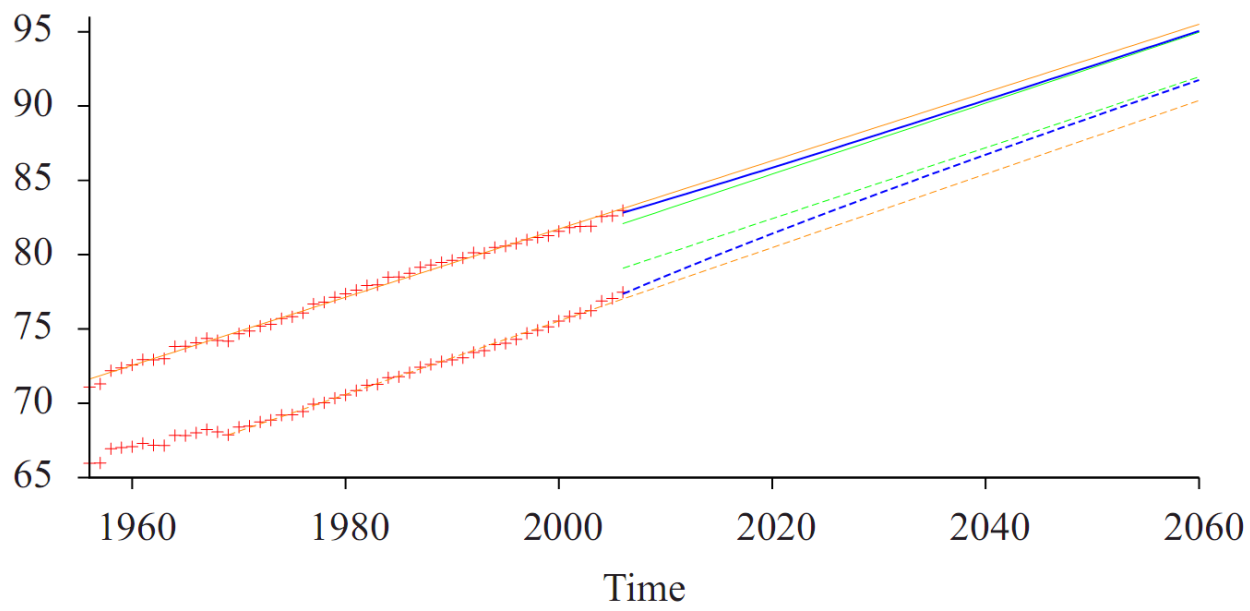
- **Common trend in life expectancies in Europe**



- **Projection approach:**
  1. Extrapolate life expectancies for total population (coherent for males and females)
  2. Determine life expectancy extrapolation for individual population relative to total population
  3. If appropriate: Modification of extrapolations to account for model uncertainty, margin,...
  4. Fit period parameters such that these life expectancy extrapolations are met

Projection of Period Parameters (ctd.)

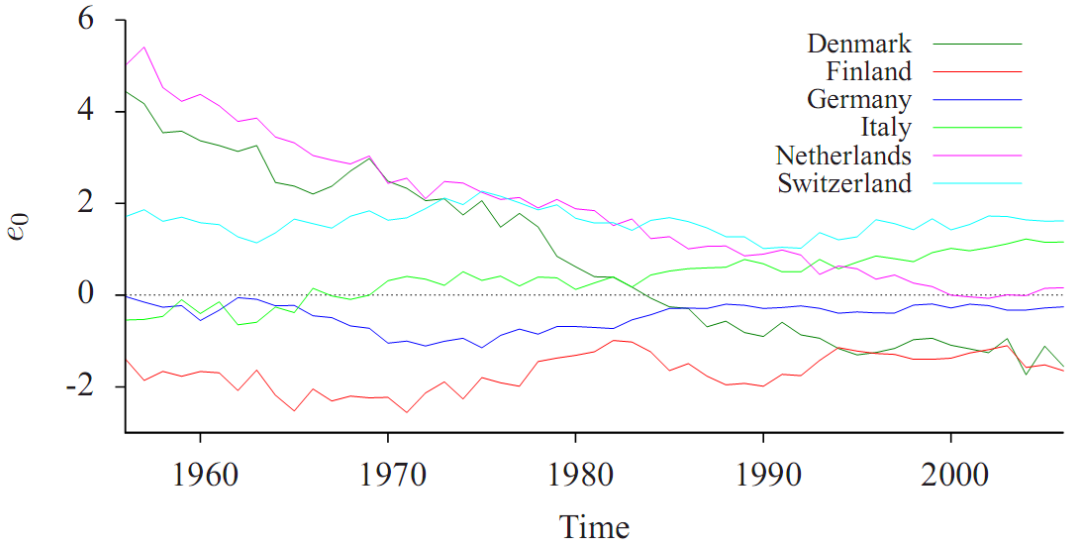
- **Life expectancy extrapolations for male and female total populations**



- Long-term trend according to average of linear historical trends for males and females
- Difference in life expectancies has been decreasing from the mid-1990's
  - Extrapolation of this trend in the short run
  - Convergence in lifestyles, e.g. comsumption of tobacco/alcohol, employment
  - Luy (2002): Difference in life expectancies between nuns and monks is only about 1 year
- Long-term difference in life expectancies of 3 years

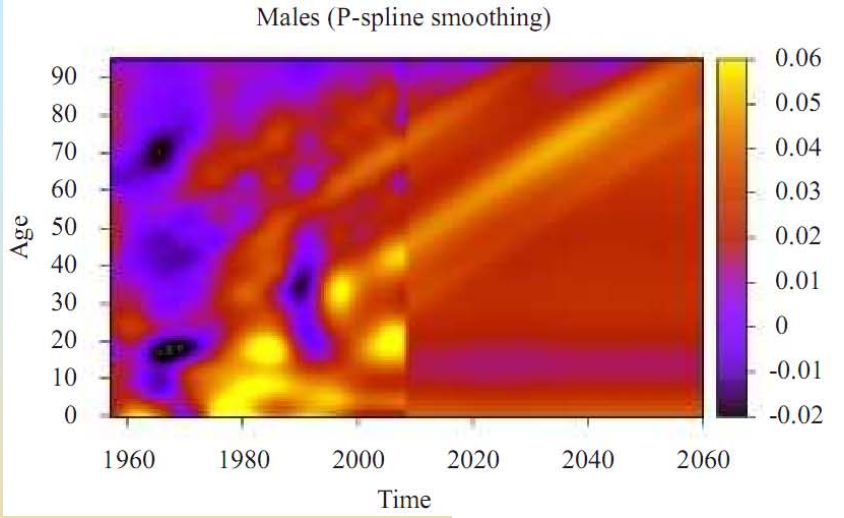
Projection of Period Parameters (ctd.)

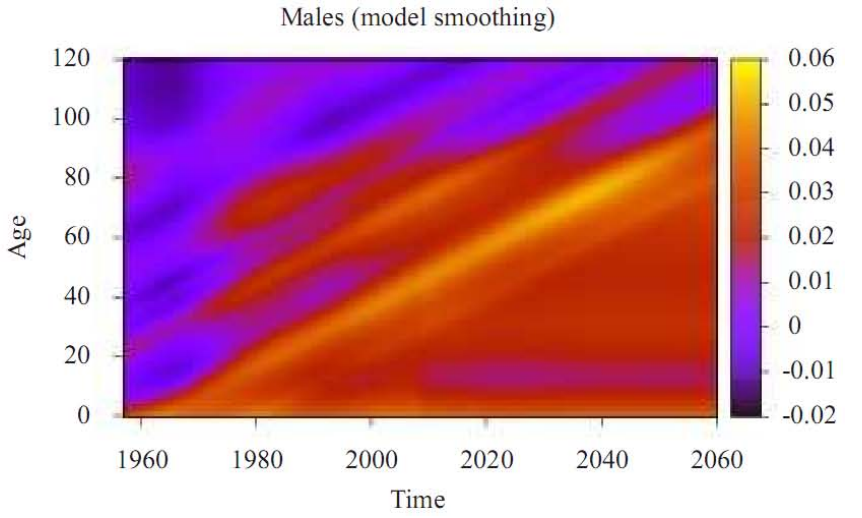
- **Life expectancies for individual populations**

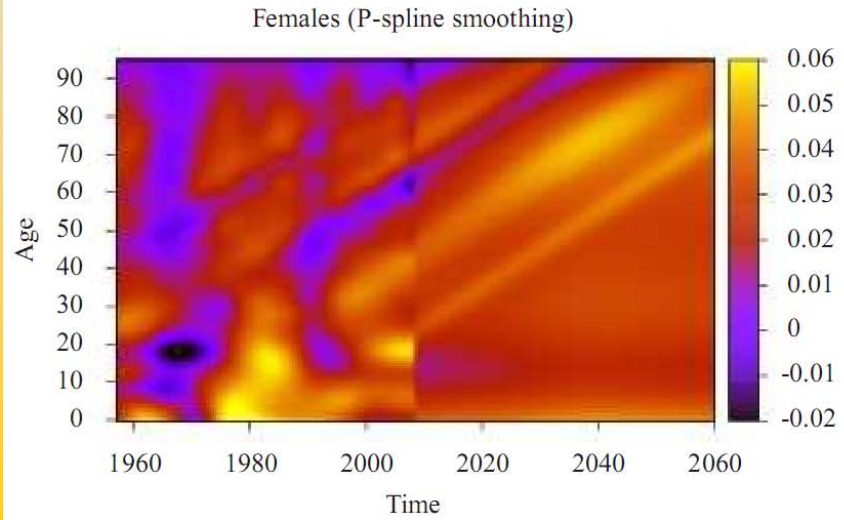


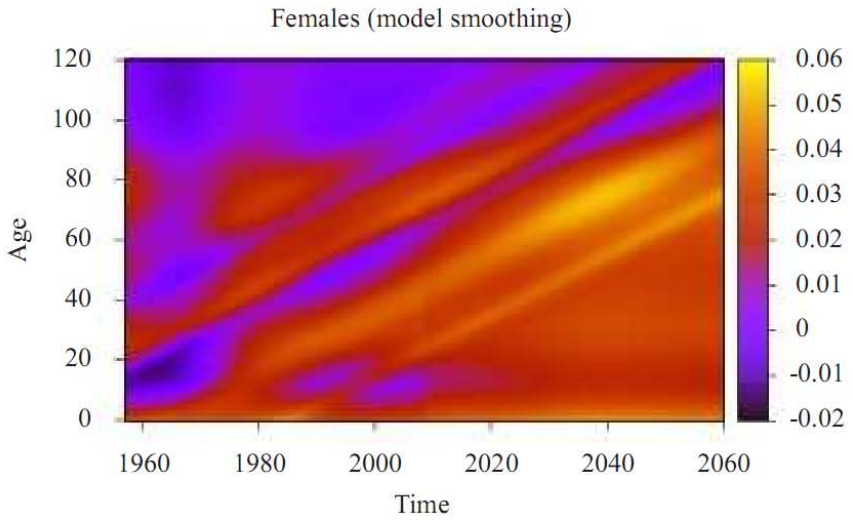
- Extrapolation according to total population not always appropriate, e.g. for Switzerland
  - Assumption of a higher life expectancy also in the long run
- Individual life expectancy projections for Italy and Denmark would not be coherent/plausible
  - A leveling-off at about the current life expectancy difference or a convergence to life expectancies of the total population seems more plausible
- For German males, life expectancies have been about 0.3 years below average in recent decades
  - Projection of individual life expectancies by downward shift by 0.3 years (0.5 years for females)

Projection for Germany









Contact Details

Matthias Boerger

Institute of Insurance, Ulm University & Institute for Finance and Actuarial Sciences (ifa), Ulm
Helmholtzstraße 22, 89081 Ulm, Germany
Phone: +49 731 50-31257, Fax: +49 731 50-31239
Email: m.boerger@ifa-ulm.de